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2.6

1) a) $y = 0$

$x - 0.04x^2 = 0$

$x(1 - 0.04x) = 0$

$x = 0$ or $1 - 0.04x = 0$

$0.04x = 1$

$x = \frac{1}{0.04} = 25$

∴ 25 meters

b) $y = x - 0.04x^2$

$y = -0.04x^2 + x$

$y = -0.04(x^2 - 25x)$

$(x - 12.5)^2 - 156.25$

$y = -0.04[(x - 12.5)^2 - 156.25]$

$y = -0.04(x - 12.5)^2 + 6.25$

c) when $x = 12.5$

$y = 6.25$

∴ maximum height is 6.25m

2) a) $P = -5.25x^2 + 73.5x - 107.5$

$P = -5.25(x^2 - \frac{73.5}{5.25}) - 107.5$

$P = -5.25(x^2 - 14x) - 107.5$

$(x - 7)^2 - 49$

$P = -5.25[(x - 7)^2 - 49] - 107.5$

$P = -5.25(x - 7)^2 + 149.75$

∴ $P = 149.75 - 5.25(x - 7)^2$

b) i) £149,750

ii) $x = 7$

$149.75 - 5.25(7 - 7)^2$

$P = 149.75$ ∴ £7

3) a) i) $\frac{2.5}{2} = 1.25$

∴ A = (1.25, -3)

ii) Roots: $x = 0$ and $x = 2.5$ → $x(x - 2.5) = 0$

$x^2 - 2.5x = 0$

↳ complete square

$(x - 1.25)^2 - 1.5625 = 0$ (x1.92)

$1.92(x - 1.25)^2 - 3 = 0$

$-3 \div -1.5625 = 1.92$

Turning point: (1.25, -3)

4) a) The model assumes that when there are no workers employed, no cars would be produced so with a constant, it wouldn't make sense. Also negative profit can still be determined as a loss, so the model would be valid.

b) $q = 0.4(1500) - 0.00008(1500)^2$

$q = 420$

c) $q = -0.00008w^2 - 0.4w$

$q = -0.00008(w^2 - 5000w)$

$q = -0.00008[(w - 2500)^2 - 6250000]$

$q = -0.00008(w - 2500)^2 - 500$

Workers: 2500

Cars: 500

5) a) 120 represents the initial height of the ball above ground level (when $t = 0$)

b) $h(t) = -4.9t^2 + 12.25t + 120$

$h(t) = -4.9(t^2 - 5/2\,t) + 120$

$h(t) = -4.9[(t - 5/4)^2 - 25/16] + 120$

$h(t) = -4.9(t - 5/4)^2 + \frac{245}{32} + 120$

$h(t) = -4.9(t - 5/4)^2 + \frac{4085}{32}$

$h(t) = \frac{4085}{32} - 4.9(t - 5/4)^2$

c) i) $\frac{4085}{32} - 4.9(t - 5/4)^2 = 0$

$\frac{4085}{32} = 4.9(t - 5/4)^2$

$\frac{4085}{32} \div 4.9 = (t - 5/4)^2$

$26.06 = (t - 5/4)^2$

$t - 5/4 = \pm\sqrt{26.06}$

$t - 5/4 = \pm 5.104$

$\therefore t = 5/4 + 5.104 = 6.354$

$t = 5/4 - 5.104 = -3.854$ (time cannot be negative)

$\therefore$ 6.35 seconds (3 s.f)

ii) $h(t) = \frac{4085}{32} - 4.9(t - 5/4)^2$

Maximum height: $4085/32$ (1283 s.f) m

Maximum time: $5/4$ seconds (1.25 seconds)

6) a) A negative profit means that the company would be making a loss so doesn't mean that the model is invalid

b) $P = -6.25x^2 + 125x - 465$

$P = -6.25\left(x^2 - \frac{125}{6.25}x\right) - 465$

$P = -6.25(x^2 - 20x) - 465$

$P = -6.25[(x-10)^2 - 100] - 465$

$P = -6.25(x-10)^2 + 625 - 465$

$P = -6.25(x-10)^2 + 160$

$\hookrightarrow 160 - 6.25(x-10)^2$

c) $160 - 6.25(x-10)^2 > 120$

$-6.25(x-10)^2 > -40$

$(x-10)^2 < 40/6.25$

$(x-10)^2 < 6.4$

$x - 10 < \pm\sqrt{6.4}$

$x - 10 < \pm 2.53$

$x < 12.53$

$x > 7.48$

d) i) £160,000

ii) $(x-10)^2 = 0$ ∴ selling price is £10

$x = 10$

7) $8q + 8 = q^2 - 14q + 48$

$q^2 - 22q + 40 = 0$

$(q-20)(q-2) = 0$

$q = 20$ OR

$q = 2$

$0 \leq q \leq 6$ ∴ $q = 2$

$P_s = 8(2) + 8 = 16 + 8 = 24$

Market equilibrium quantity: 2 (thousand)

Price: £24