

Chapter 6 - Projectiles

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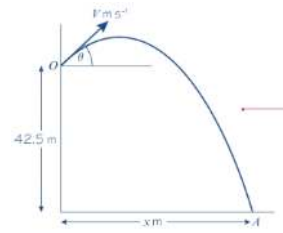
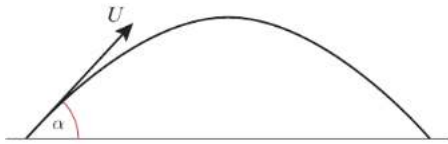
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Personal notes:



6.1 - Horizontal Projection

- Projectile is the motion of an action of a particle being acted on by a single force.



- The horizontal motion of a projectile is modelled as having constant velocity ($\mathbf{a} = \mathbf{0}$)
- The vertical motion of a projectile is modelled as having constant acceleration due to gravity ($\mathbf{a} = \mathbf{g}$)

Example

A particle is projected horizontally at 25 m s^{-1} from a point 78.4 metres above a horizontal surface. Find a) the time taken by the particle to reach the surface and b) the horizontal distance travelled in that time.



6.1 - Horizontal Projection

Example

A particle is projected horizontally with a velocity of 15ms^{-1} . Find:

- the horizontal and vertical components of the displacement of the particle from the point of projection after 3 seconds.
- the distance of the particle from the point of projection after 3 seconds.

Example

A particle is projected horizontally with a speed of $U\text{ms}^{-1}$ from a point 122.5 m above a horizontal plane. The particle hits the plane at a point which is at a horizontal distance of 90 m away from the starting point. Find the initial speed of the particle.



6.2 - Resolving Speed

Notes

Example

A particle is projected from a point on a horizontal plane with an initial velocity of 40 ms^{-1} at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

a) Find the horizontal and vertical components of the initial velocity.

Given that the vectors \mathbf{i} and \mathbf{j} are unit vectors acting in a vertical plane, horizontally and vertically respectively,

b) express the initial velocity as a vector in terms of \mathbf{i} and \mathbf{j} .

Example

A particle is projected with velocity $U = (3\mathbf{i} + 5\mathbf{j})\text{ms}^{-1}$, where \mathbf{i} and \mathbf{j} are the unit vectors in the horizontal and vertical directions respectively. Find the initial speed of the particle and its angle of projection.



6.3 - Projection at any angle

Notes

- Range is the horizontal distance travelled
- At the greatest height

Example

A particle P is projected from a point O on a horizontal plane with speed 28 ms^{-1} and with angle of elevation 30° . After projection, the particle moves freely under gravity until it strikes the plane at a point A . Find:

- a) the greatest height above the plane reached by P b) the time of flight of P c) the distance OA



6.3 - Projection at any angle

Example

A particle is projected from a point O with speed $V \text{ m s}^{-1}$ and at angle of elevation of θ , where $\tan \theta = \frac{4}{3}$. The point O is 42.5 m above a horizontal plane. The particle strikes the plane at a point A , 5 s after it is projected.

- a) Show that $V = 20$ b) Find the distance between O and A .

Example

A particle is projected from a point O with speed 35 m s^{-1} at an angle of elevation of 30° . The particle moves freely under gravity. Find the length of time for which the particle is 15 m or more above O .



6.4 - Projectile motion formulae

Notes

Recall speed $U\text{ms}^{-1}$ projected at an angle θ can be resolved into:

- x - direction: $u_x = U\cos\theta$

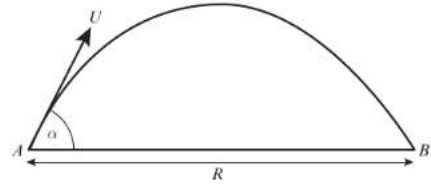
- y - direction: $u_y = U\sin\theta$

Example

A particle is projected from a point on a horizontal plane with an initial velocity U at an angle α above the horizontal and moves freely under gravity until it hits the plane at point B . Given that the acceleration due to gravity is g , find expressions for:

a) the time of flight, T

b) the range, R , on the horizontal plane.



6.4 - Projectile motion formulae

Example

A particle is projected from a point with speed U at an angle of elevation α and moves freely under gravity. When the particle has moved a horizontal distance x , its height above the point of projection is y .

a) Show that $y = x \tan \alpha - \frac{gx^2}{2U^2} (1 + \tan^2 \alpha)$.

A particle is projected from a point O on a horizontal plane, with speed 28 ms^{-1} at an angle of elevation α . The particle passes through a point B , which is at a horizontal distance of 32 m from O and at a height of 8 m above the plane.

b) Find the two possible values of α , giving your answers to the nearest degree.



6.4 - Projectile motion formulae

Exam/Practice

A girl playing volleyball on horizontal ground hits the ball towards the net 9 m away from a point 1.5 m above the ground. The ball moves in a vertical plane which is perpendicular to the net. The ball just passes over the top of the net, which is 2.4 m above the ground, as shown in the diagram. The ball is modelled as a particle projected with initial speed $U \text{ m s}^{-1}$ from point O , 1.5 m above the ground at an angle α to the horizontal.

a) By writing down expressions for the horizontal and vertical distances from O to the ball, t seconds after it was hit, show that when the ball passes over the net

$$0.9 = 9 \tan \alpha - \frac{81g}{2U^2 \cos^2 \alpha} \quad \text{(6 marks)}$$

b) Given that $\alpha = 30^\circ$, find the speed of the ball as it passes over the net. (6 marks)

