Chapter 11 - Variable Acceleration

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Personal notes:

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11.1 - Functions of time

Notes

Example

A particle is moving on a straight line from a point. The displacement, s metres, moved over t seconds is given as $s = 2t - \sqrt{t}$

a) Find the displacement of the particle after 4 seconds.
b) Find the time when the particle returns to the point.

11.1 - Functions of time

Example

A body moves in a straight line. Given that $v = 2t^2 - 16t + 24$, where v is the velocity the body moves and t is the time taken.

a) Find the initial velocity of the body.
b) Find the values of t when the body is instantaneously at rest.
c) Find the time(s) when the velocity is $64ms^{-1}$.
d) Find the greatest speed between the interval $0 \leq t \leq 5$.

Practice Q1

A particle moves in a straight line such that its velocity, $v\ ms^{-1}$, at time t seconds is given by $v = 3 + 5t - t^2$ for $t \geq 0$. Find:

a) the velocity of the particle when $t = 1$
b) the greatest speed of the particle in the interval $0 \leq t \leq 4$
c) the velocity of the particle when $t = 7$ and describe the direction of motion of the particle at this time.

11.2 - Using Differentiation

Notes (Moving off from a traffic light to another)

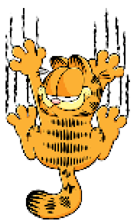
Example

A particle is moving horizontally. The displacement s from the original point is given as $s = t^3 - 3t^2 + 6$.

a) Find the velocity after 3 seconds.
b) The value of t for which the particle is instantaneously at rest.
c) Find the acceleration of the particle when $t = 5$.

Practice Q1

Garfield's, the cat, displacement from a house, in metres, is $t^3 - \frac{3}{2}t^2 - 36t$ where t is in seconds.



(a) Determine the velocity of the cat when $t = 2$.
(b) At what time will the cat be instantaneously at rest?
(c) What is the cat’s acceleration after 5 seconds?

11.3 - Maxima and Minima

Notes

- At any turning point of a graph, the gradient is 0.

- At maximum distance :

- At max./min. velocity :

Example

A child is playing yo-yo. The yo-yo travels vertically in a straight line then returns to the child's hand. The distance travelled by the yo-yo, s metres, is given by $s = 0.6t + 0.4t^2 - 0.2t^3$, where $0 \leq t \leq 3$.

a) Justify the restriction $0 \leq t \leq 3$.
b) Find the maximum distance of the yo-yo from the child's hand. (Try sketch the graph)

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11.3 - Maxima and Minima

Practice Q1

Jerry escapes from Tom and its velocity as it speeds away from Tom, is $t^3 - 16t^2 + 64t$ (in ms^{-1}), and maintains this velocity at the point where it would start slowing down.

a) When does Jerry reach its maximum velocity?
b) What is this maximum velocity?

Exam Practice (M2 June 2013 Q3)

A particle *P* moves on the x-axis. At time *t* seconds the velocity of *P* is $v\ ms^{-1}$ in the direction of x increasing, where

$$v = 2t^2 - 14t + 20, \qquad t \geq 0$$

a) Find the times when *P* is instantaneously at rest **(3 marks)**
b) the greatest speed of *P* in the interval $0 \leq t \leq 4$ **(5 marks)**

11.4 - Using Integration

Notes

- Displacement (s), velocity (v) and acceleration (a) can be expressed as a function of t (i.e. f(t))
- From previous lesson, variable acceleration causes differential relationship between the three variables (s,v,a).

Example

A particle is moving on the x-axis. At time $t = 0$, the particle is at the point where $x = 5$.
The velocity of the particle at time t seconds (where $t \geq 0$) is $(6t - t^2)$ ms^{-1}. Find:
a) An expression for the displacement of the particle from O at time t seconds.
b) The distance of the particle from its starting point when $t = 6$.

Example

A particle travels in a straight line such that its acceleration, $a\ ms^{-2}$, at time t seconds is given by $a = 6t + 2$. When $t = 2$ seconds, the displacement, s, is 10 metres and when $t = 3$ seconds the displacement is 38 metres.
a) Find the displacement when $t = 4$ seconds
b) Find the velocity when $t = 4$ seconds

11.4 - Using Integration

<u>Practice Q1</u>

A particle travels in a straight line. After t seconds its velocity, $v\ ms^{-1}$, is given by $v = 6 - 3t^2$, $t \geq 0$. Find the distance travelled by the particle in the first three seconds of its motion.
(Hint: Draw a graph first)

<u>Exam Practice (M2 June 2015 Q6)</u>

A particle *P* moves on the positive x-axis. The velocity of *P* at time *t* seconds is $(2t^2 - 9t + 4)ms^{-1}$.
When $t = 0$, *P* is 15 m from the origin.

a) Find the values of *t* when *P* is instantaneously at rest **(3 marks)**
b) Find the acceleration of *P* when *t* = 5 **(3 marks)**
c) Find the total distance travelled by *P* in the interval $0 \leq t \leq 5$ **(5 marks)**

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11.5 - Constant Acceleration Formulae

In Chapter 9, we work out the various $suvat$ formulae by using a velocity-time graph.
But it's also possible to derive all of these using integration, provided that we consider that **acceleration is constant**.

Example
Given a body moves with constant acceleration. The initial velocity is $u\ ms^{-1}$, and initial displacement is 0m.
a) Prove $v = u + at$
b) Prove $s = ut + \frac{1}{2}at^2$

Example
A particle moves in a straight line with constant acceleration. The initial velocity of particle is $5ms^{-1}$ and after 2 seconds it is moving with velocity $13ms^{-1}$.
a) Find the acceleration of the particle.
b) Without making use of the kinematics formulae, show that the displacement, s m, of the particle from its starting position is given by $s = pt^2 + qt + r, \quad t \geq 0$, where p, q and r are constants to be found.

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