Chapter 11 - Vectors

11.1 - Basic vectors - Pg. 2 - 3

11.2 - Representing vectors - Pg. 4 - 5

11.3 - Magnitude and direction - Pg. 6 - 7

11.4 - Position vectors - Pg. 8

11.5 - Solving geometric problems - Pg. 9 - 12

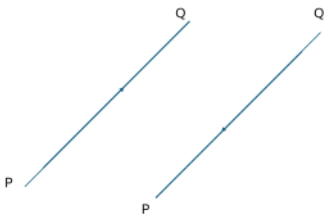
11.6 - Modelling with vectors - Pg. 13 - 14

Personal notes:

11.1 - Basic vectors

Notes

- A vector has *direction* and *magnitude*.
  - Opposite to vector is scalar (only *magnitude*)



Example

Given $\overrightarrow{AB} = a$ and $\overrightarrow{BC} = b$. Find $\overrightarrow{AC}$.

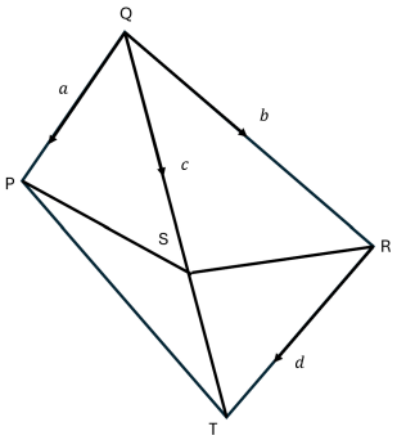
Example

Given $\overrightarrow{PQ} = a$. M is a point on PQ that divides PQ in the ratio 2 : 1.
Find $\overrightarrow{QM}$.

Example/Practice Q1

Given $\overrightarrow{QP} = a$, $\overrightarrow{QR} = b$, $\overrightarrow{QS} = c$, $\overrightarrow{RT} = d$.
Find

a) $\overrightarrow{PS}$
b) $\overrightarrow{RP}$
c) $\overrightarrow{PT}$
d) $\overrightarrow{TS}$



11.1 - Basic vectors

Notes

- If two lines are *parallel*, then one vector would be a *multiple* of another.
- If two lines are *equal in length*, then their *magnitudes* are equal.

Example

Given $\overrightarrow{PQ} = 2a + 3b$, $\overrightarrow{AB} = 6a + 9b$.

Prove $\overrightarrow{PQ}$ and $\overrightarrow{AB}$ are parallel.

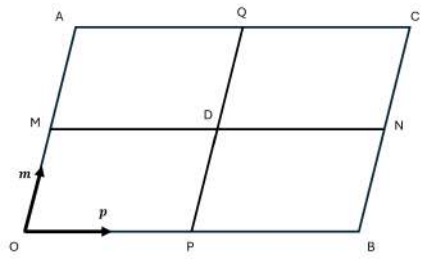
Practice Q2

OACB is a parallelogram, *M, Q, N* and *P* are the midpoints of *OA, AC, BC* and *OB* respectively. Vectors **p** and **m** are equal to $\overrightarrow{OP}$ and $\overrightarrow{OM}$ respectively. Express in terms of **p** and **m**.

a) $\overrightarrow{OA}$ b) $\overrightarrow{OB}$ c) $\overrightarrow{BN}$ d) $\overrightarrow{DQ}$

e) $\overrightarrow{OD}$ f) $\overrightarrow{MQ}$ g) $\overrightarrow{OQ}$ h) $\overrightarrow{AD}$

i) $\overrightarrow{CD}$ j) $\overrightarrow{AP}$ k) $\overrightarrow{BM}$ l) $\overrightarrow{NO}$



Practice Q3

$ABCD$ is a trapezium with *AB* parallel to *DC* and *DC* = 3*AB*. *M* divides *DC* such that *DM : MC* = 2 : 1. $\overrightarrow{AB} = \boldsymbol{a}$ and $\overrightarrow{BC} = \boldsymbol{b}$. Find in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:

a) $\overrightarrow{AM}$

b) $\overrightarrow{BD}$

c) $\overrightarrow{MB}$

d) $\overrightarrow{DA}$

11.2 - Representing vectors

Notes

There are two ways to represent vectors:

- Column vector form

- $i - j$ vector form

Column vector:

$i - j$ form:

- The unit vector $\boldsymbol{i}$ represents
- The unit vector $\boldsymbol{j}$ represents

Example

Given $a = 6i - 9j$, $b = 12i + 6j$. Find

a) $\frac{1}{2}a$

b) $2a - 5b$

Practice Q1

Given that $\boldsymbol{a} = \begin{pmatrix} \mathbf{9} \\ \mathbf{7} \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} \mathbf{11} \\ \mathbf{3} \end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix} \mathbf{-8} \\ \mathbf{-1} \end{pmatrix}$.

Find

a) 5a b) $-\frac{1}{2}c$ c) $2b + 2c - 3a$

d) $\frac{1}{2}a + \frac{1}{2}b$

11.2 - Representing vectors

Example

Given that $\boldsymbol{c} = 3i + 4j$ and $\boldsymbol{d} = i - 2j$. Find

a) λ if $\mathrm{c} + \lambda\mathrm{d}$ is parallel to $i + 2j$

b) μ if $\mu\mathrm{c} - \mathrm{d}$ is parallel to $-4i + 3j$

Practice Q2

Given that $\boldsymbol{c} = 2i + j$ and $\boldsymbol{d} = i - 5j$. Find

a) λ if $\mathrm{c} + \lambda\mathrm{d}$ is parallel to $2i + 3j$

b) μ if $d - \mu c$ is parallel to $-2i + 3j$

Exam Practice

Given that the resultant of the vectors $\boldsymbol{a} = 2p\boldsymbol{i} - 5\boldsymbol{j}$ and $\boldsymbol{b} = 6\boldsymbol{i} - 3p\boldsymbol{j}$ is parallel to the vector $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j}$**.**

a) find the value of p. **(4 marks)**

b) find the resultant of the vectors **a** and **b**. **(2 marks)**

11.3 - Magnitude and direction

Notes

- Vectors only tell direction on the surface.
  e.g. $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$ and $\begin{pmatrix} 10 \\ -12 \end{pmatrix}$ have the same distance

  because…

- Distance of a vector could be found as the ***magnitude*** of the vector, denoted as $|\square|$.

- If $a = \begin{pmatrix} x \\ y \end{pmatrix}$, then $|a| =$

Example

Given $a = 7i + 24j$ and $b = -5i - 12j$.

a) Find $|a|$ and $|b|$.
b) Find the unit vector of a.
c) Find the angle between a and the positive $x - axis$.
d) Find the angle between b and the positive $y - axis$
e) Given vector c has a magnitude 100 and makes an angle of $30°$ with i. Find c.

11.3 - Magnitude and direction

Practice Q1

Given $\boldsymbol{a} = 2i + 3j$, $\boldsymbol{b} = 3i - 4j$ and $\boldsymbol{c} = 5i - j$. Find the magnitude of

a) $\boldsymbol{a} + \boldsymbol{b}$

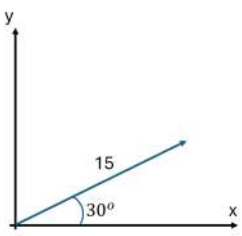
b) $2a - c$

c) Find the unit vectors of $\boldsymbol{a}$ and **b**.

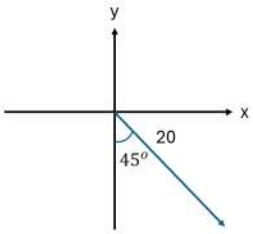
Practice Q2

Write the following in $i - j$ vector form:

a)



b)



Exam Practice

Given that $\left|2i - kj\right| = 2\sqrt{10}$, find the exact value of k.

(3 marks)

11.4 - Position vectors

Notes

- Recall: Vectors describe the direction and the magnitude describes the distance travelled by the vectors.
- Vectors could also be used to describe *location or position.*

- ***Position vectors*** are vectors giving the position of a point relative to a fixed point (usually origin).

Example

Given point A and B have coordinates $(3,2)$ and $(-1,9)$ respectively.

a) Find the position vector of A.
b) Find the position vector of B.
c) Find the vector $\overrightarrow{AB}$.

Given $\overrightarrow{BC} = 3i + 4j$.

d) Find the position vector of C.

Practice Q1

Given that the point *A* has position vector $4i - 5j$ and the point *B* has position vector $6i + 3j$.

a) Find the vector $\overrightarrow{AB}$. **(2 marks)**
b) Find $\left|\overrightarrow{AB}\right|$ giving your answer as a simplified surd. **(2 marks)**

11.5 - Solving geometric problems

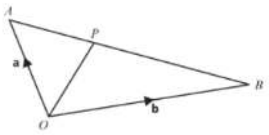
Notes

- When a vector is parallel to another vector, we can express it as *a multiple of each other.*

- E.g. If $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$ and given $\overrightarrow{AB} = a$, then $\overrightarrow{CD} =$

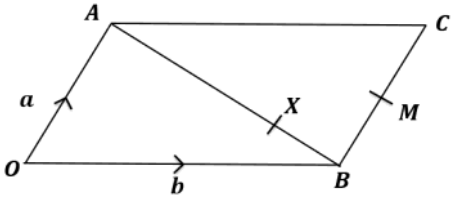
Example/Practice Q1

The points *A* and *B* have position vectors **a** and **b** respectively, as shown in the diagram. The point *P* divides *AB* in the ratio 1 : 2. Find the position vector of *P*.



Example*

Given OABC is a parallelogram and X is a point on AB such that AX : XB = 3 : 1. M is the midpoint of BC. Show that $\overrightarrow{XM}$ is parallel to $\overrightarrow{OC}$.

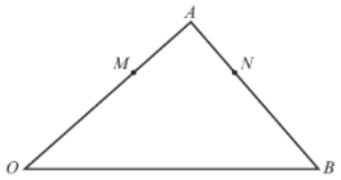


11.5 - Solving geometric problems

Example**

OAB is a triangle. $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$. The point *M* divides *OA* in the ratio 3 : 2. Given *MN* is parallel to *OB*:

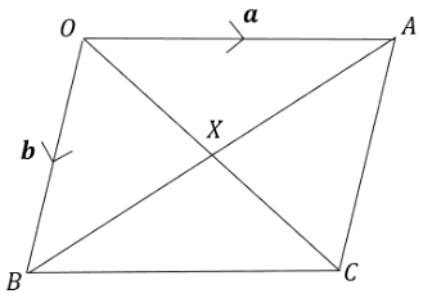
a) Express the vector $\overrightarrow{ON}$ in terms of **a** and **b**.
b) Show that *AN : NB* = 2 : 3



11.5 - Solving geometric problems

Example***

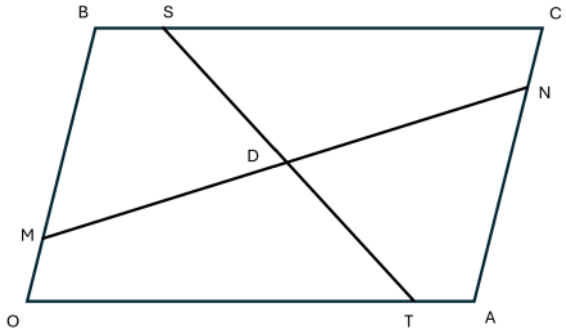
$OACB$ is a parallelogram, where $\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$.
The diagonals OC and AB intersect at a point X.
Prove that the diagonals bisect each other.



11.5 - Solving geometric problems

Practice Q2

OACB a parallelogram. $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.
The points *M, S, N* and *T* divide *OB, BC, CA* and *AO* in the ratio $1:4$ respectively.
The lines *ST* and *MN* intersect at the point *D*.



a) Express $\overrightarrow{MN}$ in terms of **a** and **b**.
b) Express $\overrightarrow{ST}$ in terms of **a** and **b**.
c) Show that the lines *MN* and *ST* bisect one another.

11.6 - Modelling with vectors

Example

A car is moving with velocity $(6i + 12j)kmh^{-1}$ from the origin. Find the speed the car is travelling.

Example

Mr. Fan starts from O and walks 15km on a bearing of $120°$ to A. Then he walks 9km on a bearing of $240°$ to B. After that he returns from B to O directly.

a) Find the position vector of A relative to O.
b) Find $\left|\overrightarrow{OB}\right|$.
c) Find the bearing from O to B.
d) Find the position vector of B relative to O.

11.6 - Modelling with vectors

Practice Q1

A girl walks 2km due east from a fixed point *O* to *A*, and then 3km due south from *A* to *B*. Find:

a) The total distance travelled
b) The position vector of *B* relative to *O*
c) $\left|\overrightarrow{OB}\right|$
d) The bearing of *B* from *O*

(Hint: Draw a diagram to help you)

Practice Q2

Two forces, $\boldsymbol{E}$ and $\boldsymbol{F}$, are given by the vectors $\boldsymbol{E} = (3i - 4j)N$ and $\boldsymbol{F} = (pi + qj)N$. The resultant force, $\boldsymbol{R} = \boldsymbol{E} + \boldsymbol{F}$ acts in a direction which is parallel to the vector $(2i - j)$.

a) Find the angle between **R** and the vector i.
b) Show that $p + 2q = 5$
c) Given that $p = 1$, find the magniture of **R.**