Chapter 13 - Integration

Personal notes:

13.1 - Integration x^n

Notes

- Integration is the opposite of differentiation.

- However, when differentiating $y = 2x^3 + 1$ and $y = 2x^3 - 5$....
  Both get the same derivative: $\frac{dy}{dx} = 6x^2$
- Technically, the original function could have had any constant, as it disappears upon differentiation, therefore we have to _____ after integration.

- If $\frac{dy}{dx}$ or $f'(x) = x^n$, then $f(x) =$

Examples

Find y when

a) $\frac{dy}{dx} = x^4$

b) $\frac{dy}{dx} = 3x^5$

c) $\frac{dy}{dx} = -2x^{\frac{1}{3}}$

d) $\frac{dy}{dx} = \frac{3}{\sqrt{x}}$

Example

Given $f'(x) = 5x + 4x^{-3} - \frac{3}{\sqrt[3]{x}}$, find $f(x)$.

13.1 - Integration x^n

Practice Q1

Find $f(x)$ when

a) $f'(x) = 2x + 7$
b) $f'(x) = x^2 - 1$
c) $f'(x) = \frac{2}{x^7}$
d) $f'(x) = \sqrt[4]{x} + 2x$
e) $f'(x) = 10x^{\frac{3}{4}} - 3x^{-2}$

Challenge

Find y when $\frac{dy}{dx} = \left(2\sqrt{x} - x^2\right)\frac{(3+x)}{x^5}$

(Hint: Expand brackets)

13.2 - Indefinite integrals

Notes

- We can use the symbol $\int dx$ to represent the process of *integration*.

Differentiation	Integration
$y = f(x)$ $\frac{dy}{dx} = f'(x)$	$\int \frac{dy}{dx} dx = \int f'(x)\, dx$ $y = f(x)$

- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

Example

a) Find $\int (x^{-\frac{5}{2}} + 2) dx$

b) Find $\int \left(4t^{-2} - 1\right) dt$

c) Find $\int \left(px^3 + q^{-2}\right) dx$, where p and q are constants

Practice Q1

a) Find $\int (2x^{\frac{3}{2}} + 5x) dx$
b) Find $\int \left(-3a^{-4} + 5\right) da$
c) Find $\int \left(s^3 + rt^{-2}\right) dt$, where r and s are constants

13.2 - Indefinite integrals

Slightly Harder examples

a) $\int (\frac{2}{x^3} - 3\sqrt{x}\,)dx$

b) $\int x(x^2 + \frac{2}{x})\,dx$

c) $\int ((2x)^2 + \frac{\sqrt{x}+5}{x^2})dx$

Practice Q2

a) $\int (\frac{2+x}{x^3} + 3)dx$

b) $\int \frac{(2x+1)^2}{\sqrt{x}}\,dx$

c) $\int \sqrt{x}\left(\sqrt{x}+3\right)^2\,dx$

Exam Practice (C1 May 2014 Q4b)

Given that $y = 2x^5 + \frac{6}{\sqrt{x}}$, $x > 0$, find in the simplest form:

(b) $\int y\,dx$ **(3 marks)**

13.3 - Finding functions

Starter

a) $\frac{d}{dx}(4x^3 + 16x^{\frac{1}{2}} + 20x + 6)$

b) $\frac{d}{dx}(4x^3 + 16x^{\frac{1}{2}} + 20x - 10)$

c) $\frac{d}{dx}(4x^3 + 16x^{\frac{1}{2}} + 20x + 100)$

What do you notice?

->

Notes

- To find the "+c" value,

Example

The curve with equation $y = f(x)$ passes through $(2,3)$. Given that $f'(x) = 2x^2 - 5$, find the equation of the curve.

Practice Q1

The curve with equation $y = f(x)$ passes through $(4,5)$. Given that $f'(x) = \frac{x^2-2}{\sqrt{x}}$, find the equation of the curve.

13.3 - Finding functions

Exam Practice (C1 May 2014 Q10)

A curve with equation $y = f(x)$ passes through the point (4, 25).

Given that $f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1$, $x > 0$

a) find $f(x)$, simplifying each term **(5 marks)**

b) find an equation of the normal to curve at the point (4, 25).

Give your answer in the form $ax + by + c = 0$, where *a, b* and *c* are integers to be found.

(5 marks)

13.4 - Definite integrals

Notes

- We can work out a numerical answer of an integral between two **limits**. This is called **definite integral**.

Given $f'(x) = 4x^3$

Indefinite Integral	Definite Integral (between $x = 1$ and $x = 5$)
$f(x) = \int 4x^3 \, dx$ $f(x) = \frac{4x^4}{4} + c$ $f(x) = x^4 + c$	$f(x) = \int_1^5 4x^3 \, dx$ $f(x) = \left[x^4\right]_1^5$ $f(x) = (5^4) - (1^4) = 624$

- $\int_a^b f'(x) \, dx =$

Example

Evaluate $\int_0^1 (\sqrt[3]{x} - 4) \, dx$

Practice Q1

a) $\int_2^5 (\frac{2}{x^3} + 3x) dx$

b) $\int_4^9 \left(\sqrt{x} - \frac{6}{x^2}\right) dx$

13.4 - Definite integrals

Example

Given that P is a constant and $\int_1^5 (2Px + 7)dx = 4P^2$, show that there are two possible values of P and find these values.

Practice Q2

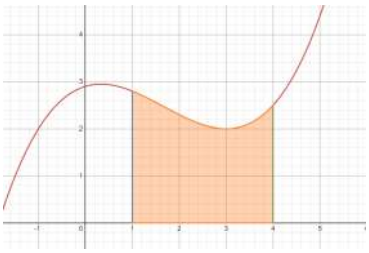
Given that A is a constant and $\int_1^4 (6\sqrt{x} - A)dx = A^2$, show that there are two possible values of P and find these values.

13.5 - Areas under curves

Notes

- The area between a positive curve, the x-axis and the lines $x = a$ and $x = b$ is given by…

  Area =



Example

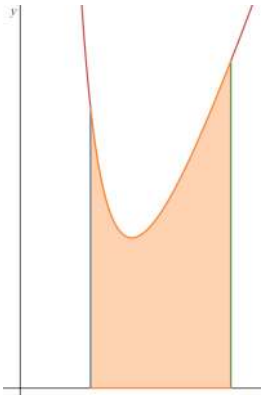
Given a curve has equation $y = 20 - x - x^2$,

a) Sketch the curve
b) Find the area of the finite region between the curve and the x-axis.

Practice Q1

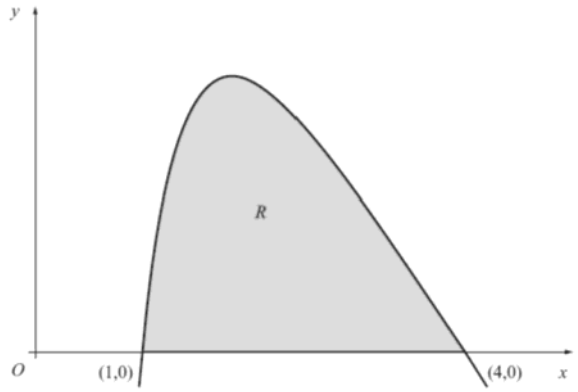
The diagram shows a sketch of the curve with equation $y = 3x + \frac{6}{x^2} - 5$, $x > 0$.

The region R is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 3$. Find the area of R.



13.5 - Areas under curves

Exam Practice (C2 Jan 2019 Q9c)



The finite region *R*, as shown, is bounded by the x-axis and the curve with equation $y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}$, $x > 0$

The curve crosses the x-axis at the points $(1,0)$ and $(4,0)$.
(c) Use integration to find the exact value for the area of *R*. **(6 marks)**

13.6 - Areas under x-axis

Starter

a) Sketch the graph of $y = x(x-1)(x-2)$
b) Work out $\int_0^2 x(x-1)(x-2)\,dx$
c) What does the answer in part (b) represent? What's wrong with the answer?

Notes

- When the area bounded by the curve and the x-axis is *below* the x-axis, the $\int y\,dx$ gives a _____.
- To work out the area of the starter above, we should

Example

Find the area of the finite region bounded the curve $y = x(x-1)(x-2)$ and the x-axis.

13.6 - Areas under x-axis

Practice Q1

Sketch the curve with equation $y = x(x-1)(x+3)$ and find the area of the finite region bounded by the curve and the x-axis.

Exam Practice (C2 May 2013 Q6)

Figure 3 shows a sketch of part of the curve *C* with equation $y = x(x+4)(x-2)$.

The curve *C* crosses the x-axis at the origin *O* and at the points *A* and *B*.

(a) Write down the x-coordinates of the points *A* and *B*. **(1 mark)**

The finite region, shown shaded in Figure 3, is bounded by the curve *C* and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3. **(7 marks)**

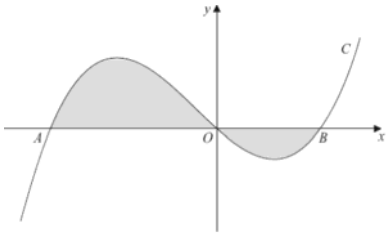


Figure 3

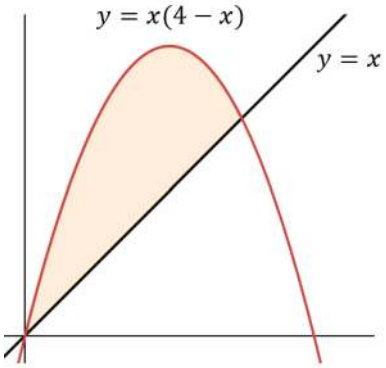
13.7 - Areas between curves and lines

Notes

- We can use definite integrals together with area of triangles to find more complicated areas (such as bounded between two graphs).

Example

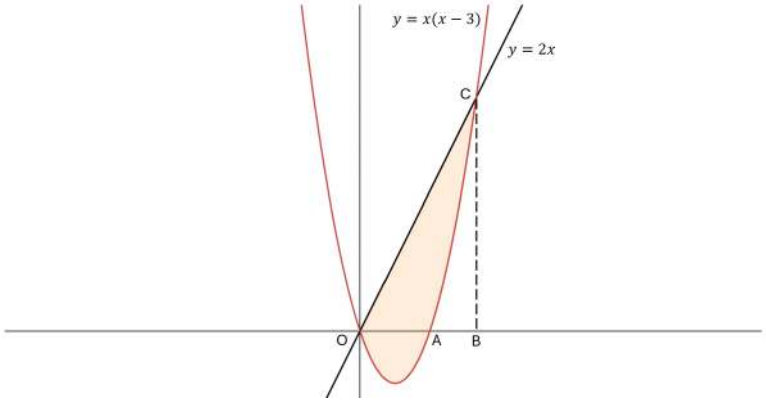
The diagram shows a sketch of part of the curve with equation $y = x(4 - x)$ and the line with equation $y = x$. Find the area of the region bounded by the curve and the line.



Example

The diagram shows a sketch of the curve with equation $y = x(x - 3)$ and the line with equation $y = 2x$.

a) Find the coordinates of points *A* and *B*.
b) Find the area of the shaded region *OAC*.



13.7 - Areas between curves and lines

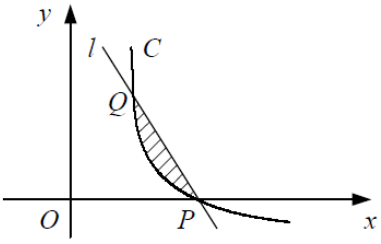
Practice Q1

The diagram shows the curve *C* with equation $y = \frac{4-x^2}{x^2}$, $x > 0$, and the straight line l.

a) Find the coordinates of the point *P* where *C* crosses the x-axis.

The line l has gradient -3 and intersects *C* at the points *P* and *Q*.

a) Find the equation of the line l.
b) Find the coordinates of the point *Q*.
c) Show that the area of the shaded region enclosed by *C* and l is $\frac{1}{2}$.



Exam Practice (C2 May 2012 Q5)

The figure shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$.

The line and the curve intersect at the points *A* and *B*, and *O* is the origin.

(a) Calculate the coordinates of *A* and the coordinates of *B*. **(5 marks)**

The shaded area *R* is bounded by the line and the curve, as shown in the figure.

(b) Calculate the exact area of *R*. **(7 marks)**