

Chapter 1 and 2 - A for Algebra

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Personal notes:



1.1 - GCSE Algebra Recap

Index laws

- $a^m \times a^n =$
- $a^m \div a^n =$
- $(a^m)^n =$
- $(ab)^n =$

Example

Simplify $(a^5)^2 \times 3a^2$

Example

Simplify $(3x^3y^4)^2$

Practice Q1

Simplify the following expressions:

- a) $7a^4 \times (3a^4)^2$
- b) $(4y^3)^3 \div 2y^3$

Expanding brackets

- If you were ever taught the "FOIL" method, cleanse it from your mind!
- Multiply each term in the first bracket by each term in the second.

Example

$(x + 1)(x + 2)(x + 3)$

Example

$x(5x - 3y)(2x - 3y + 4)$

Practice Q2

- a) $(x - 4)(x - 2)(x + 1)$
- b) $(2x - 1)^3$

BF MATHS

1.1 - GCSE Algebra Recap

Fractional and Negative Indices

- $a^{\frac{1}{m}} =$
- $a^{\frac{n}{m}} =$
- $a^{-m} =$
- $a^0 =$

Example

Simplify the following expressions:

a) $\frac{x^3}{x^{-3}}$

b) $(x^3)^{\frac{2}{3}}$

c) $(125x^6)^{\frac{1}{3}}$

d) $\left(\frac{1}{9}x^6y\right)^{\frac{1}{2}}$

Practice Q3

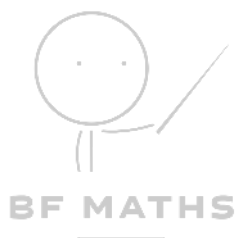
Simplify the following expressions:

a) $3x^4 \times 2x^{0.5}$

b) $\sqrt{x} \times \sqrt[3]{x}$

Challenge:

Given $2^x \times 4^y = \frac{1}{2\sqrt{2}}$. Express y as a function of x (make y the subject).



1.1 - GCSE Algebra Recap

Quadratic Equations

- A quadratic equation can be written in the form $ax^2 + bx + c = 0$, where a, b and c are real constants.
- Quadratic equations can have *one, two or no* real solutions.

Ways to deal with a quadratic equation:

- Factorising

- Completing Square

- Quadratic formula

Example

Solve the following equations:

- $x^2 - 2x = 15$
- $x^2 = 9x$

Practice Q4

Solve the following quadratics:

- $x^2 - 4x + 3 = 0$
- $x(x - 1) = 42$
- $6x^2 = 9x$

Example

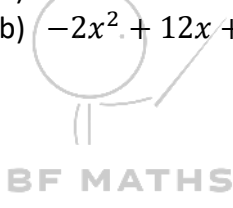
Complete the square for the following expressions:

- $x^2 + 8x$
- $2x^2 - 12x$
- $-x^2 - 14x + 20$

Practice Q5

Complete the square for the following expressions:

- $x^2 + 16x$
- $-2x^2 + 12x + 7$



2.3 - Disguised Quadratics

Disguised Quadratics

- Some equations does not look alike to normal quadratics, we have to do _____ to turn them into quadratics.

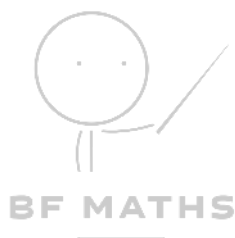
Example

Solve the following quadratics:

- $x^4 - 3x^2 - 4 = 0$
- $2x + 7\sqrt{x} + 3 = 0$

Practice Q1

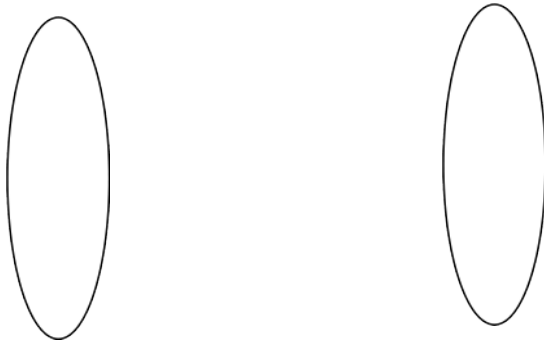
- $x^4 - 8x^2 + 16 = 0$
- $x^4 + x^2 - 6 = 0$
- $x^4 - 49 = 0$
- $4x + 7\sqrt{x} - 2 = 0$
- $4x + 4x^{\frac{1}{2}} + 1 = 0$



2.4 - Quadratic Functions and Graphs

Notes

- A function is a mathematical relationship that maps each value of a set of inputs to an output.
- A function of x is denoted as



- The **domain** of a function is
- The **range** of a function is
- The **roots** of a function are

Example

If $f(x) = x^2 - 3x$ and $g(x) = x + 5$, $x \in \mathbb{R}$

- Find $f(-4)$
- Find the values of x which $f(x) = g(x)$
- Find the roots of $f(x)$
- Find the roots of $g(x)$

Example

The function f is defined as $f(x) = x^2 + 6x - 5$, $x \in \mathbb{R}$.

- Write $f(x)$ in the form of $(x + p)^2 + q$ and state the values of p and q .
- Hence, or otherwise, find the roots of $f(x)$, leaving your answers in surd form.
- Write down the minimum value of $f(x)$ and state the value of x for which it occurs.



2.4 - Quadratic Functions and Graphs

Practice Q1

For each of the following functions, find the roots and the minimum value of $f(x)$.

a) $x^2 + 4x - 9$

b) $x^2 - 10x + 21$

Practice Q2

Find the roots of the function $f(x) = x^4 - x^2 - 6$



2.4 - Quadratic Functions and Graphs

Sketching Quadratic Graphs

- 1.
- 2.
- 3.

Finding coordinates of turning point

-
- Turning point =

Example

Sketch the graph of $y = x^2 - 5x + 4$, and find the coordinates of its turning point.



2.4 - Quadratic Functions and Graphs

Example

Sketch the graph of $y = -2x^2 + 4x - 3$. Find the coordinates of its turning point.

Practice Q3

Sketch the graph and work out the turning point of each of the following equations:

a) $y = x^2 + 4$

b) $y = -2x^2 + 5x + 3$



2.5 - Discriminant

Notes

- The discriminant of a quadratic expression =

If the quadratic equation has...			
	Two distinct real roots	One real root / Repeated root	No real root
Discriminant			
Graphical representations			

Example

Find the values of k for which $f(x) = x^2 + kx + 9$ has equal roots.

Example

Find the range of values of k such that $x^2 + 4x + k = 0$ has two distinct real solutions.

Practice Q1

For each quadratic equation, work out the discriminant and determine the number of real roots.

- $x^2 + 3x + 4 = 0$
- $2x^2 - 6x - 3 = 0$
- $x - 4 - 3x^2 = 0$
- $1 - x^2 = 0$



2.5 - Discriminant

Example/Practice Q2 (Exam question)

8. The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

(a) Find the value of p .

(4)

(b) For this value of p , solve the equation $x^2 + 2px + (3p + 4) = 0$.

(2)

Exam Practice Q3

$$x^2 + 5kx + (10k + 5) = 0$$

where k is a constant.

Given that this equation has equal roots, determine the value of k .



2.6 - Modelling with quadratics

Notes

- A mathematical model is a mathematical description of a real-life situation, simply we apply maths in real-world context, e.g. the flight path of a moth, the circular property of a running track, the trajectory of a basketball into the hoop...etc
- The key skills to do well in modelling:
 - knowing which letters are variables, which ones are constants
 - linking the given numerical or worded information to the variables

Example

A hunter stands on the top of a hill and shoots an arrow to the ground level to hunt a fox. The height, in metres, of the arrow above the ground after t seconds is modelled by the function:

$$h(t) = 5.2 + 7.35t - 4.9t^2, t \geq 0$$

- Interpret the meaning of the constant term 5.2 in the model.
- After how many seconds does the arrow hit the ground.
- Write $h(t)$ in the form $A - B(t - C)^2$, where A , B and C are constants to be found.
- Using your answer to part c or otherwise, find the maximum height of the arrow above the ground, and the time at which this maximum height is reached.

