Chapter 4 - Graphs and Transformations

Personal notes:

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4.1 - Cubic Graphs

Notes

Monomial	Binomial	Polynomial

- The degree of a polynomial with one variable is the largest power in the polynomial.

  Polynomial with degree of 6:

  Polynomial with degree of 5:

  Quartic expression:

  Cubic expression:

  Quadratic expression:

  Linear expression:

Basic steps to sketch graph

1.

2.

3.

4. Sketch

Example (Quadratic)

Sketch the graph of $y = x^2 + 5x + 6$

Practice Q1 (Quadratic)

a) Sketch the graph of $y = (x + 2)(x - 1)$

b) Sketch the graph of $y = (1 - x)(2x + 1)$

4.1 - Cubic Graphs

Notes (Cubic Graphs) : $y = ax^3 + bx^2 + cx + d$

Example (Cubic - Basic)
Sketch the graph of $y = x^3 - 2x^2 - 3x$

Example (Cubic - repeated root)
Sketch $y = (x^2 - 1)(x - 1)$

Example (Cubic - triple root)
Sketch the graph of $y = -(x - 1)^3$

Practice Q2 (Cubic - Basic)
Sketch the graph of $y = (x - 3)(x + 2)(x - 1)$

Practice Q3 (Cubic - repeated root)
Sketch $y = (2 - x)(x + 1)^2$

Practice Q4 (Cubic - triple root)
Sketch the graph of $y = (2x + 1)^3$

4.2 - Quartic Graphs

<u>Notes (Quartic graphs):</u> $\boldsymbol{y = ax^4 + bx^3 + cx^2 + dx + e}$

<u>Example (Basic)</u>
Sketch the graph of $y = (x+1)(x-3)(x+2)(x+6)$

<u>Practice (Basic)</u>
Sketch the graph of $y = (x+2)(x+1)(x-3)(x-2)$

<u>Example (Double root)</u>
Sketch $y = (x+1)^2(x^2+12x+36)$

<u>Practice Q1 (Double root)</u>
Sketch $y = -(x-4)^2(x+2)^2$

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4.2 - Quartic Graphs

Example (Triple root)
Sketch $y = -(x+1)(x-4)(x^2-8x+16)$

Practice Q2 (Triple root)
Sketch $y = (x+2)^3(2x-3)$

Example (Quadruple root)
Sketch $y = (x-5)^4$

Practice Q3 (Quadruple root)
Sketch $y = (3x-4)^4$

4.3 - Reciprocal Graphs

Notes

- Why does a fraction with denominator as 0 in the calculator give "Maths error?"
- Technically, they are ***"undefined".***
- To represent this feature on graph, we use ___________.

Basic reciprocal graph prototypes:

Practice Q1

Sketch the following graphs on different axes:

a) $y = \frac{2}{x}$ b) $y = -\frac{3}{x^2}$ c) $y = -\frac{1}{x}$

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4.4 - Points of Intersection

Notes

- The points of intersection of two curves, $y = f(x)$ and $y = g(x)$, can be found by solving $f(x) = g(x)$.

- Points of intersection/two lines *meet* =>

- Graphically, the number of intersection points = the number of real solutions to the equation $f(x) = g(x)$.

Example
Find the coordinates of points of intersection of $y = x(x-3)$ and $y = x^2(1-x)$.

Practice Q1
a) Sketch the two curves on the same axes: $y = \frac{4}{x^2}$ and $y = x^2(x-3)$
b) Show that the equation $\frac{4}{x^2} = x^2(x-3)$ can be rearranged to $x^4(x-3) - 4 = 0$
c) Using your sketch, state, with a reason, the number of real solutions to the equation $x^4(x-3) - 4 = 0$.

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4.5 - Translating Graphs

Starter

Let $f(x) = x^2 + 2x + 1$

Find the value of

i) $f(1)$
ii) $f(2)$
iii) $f(3)$
iv) $f(1) + 6$
v) $f(2) + 9$

Find the expression of

vi) $f(x + 1)$

vii) $f(x) + 1$

Notes

- $f(x + a)$ represents
- $f(x) + a$ represents

Example

a) Sketch $g(x) = x(x - 2)$
b) Sketch $g(x + 3)$
c) Sketch $g(x) - 5$

Practice Q1

a) Sketch $f(x) = \frac{1}{x}$ and write down the equation(s) of the asymptote(s)
b) Sketch $f(x - 4)$ and write down the equation(s) of the asymptote(s)
c) Sketch $f(x) + 6$ and write down the equation(s) of the asymptote(s)

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4.5 - Translating Graphs

Example (Transformed graphs)

a) Sketch $y = x^2 + 3$

b) Sketch $y = \frac{2}{x+1}$

Practice Q2 (Transformed graphs)

a) Sketch $y = -x^2 - 4$

b) Sketch $y = \frac{3}{x} + 2$

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4.6 - Stretching and reflecting graphs

Recap

- $f(x+a)$ represents a horizontal translation of $-a$.
- $f(x)+a$ represents a vertical translation of a.

Notes

- $f(ax)$ represents
- $af(x)$ represents
- $f(-x)$ represents
- $-f(x)$ represents

Example

Given that $f(x) = (x-3)^2$

a) Sketch $y = f(2x)$

b) Sketch $y = 3f(x)$

Example

Given $f(x) = x(x+3)(x-2)$

a) Sketch $f(-x)$

b) Sketch $-f(x)$

Practice Q1

Given that $f(x) = (2-x)^2$, sketch the graph of:

a) $f(3x)$ b) $\frac{1}{2}f(x)$ c) $f(-x)$ d) $-f(x)$

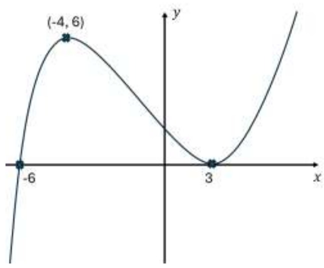
Practice Q2

The diagram shows the graph of $y = f(x)$. The graph crosses the x-axis at $(-6,0)$ and has turning points $(-4,6)$ and $(3,0)$.

a) Sketch the graph of $y = -f(x)$, state clearly the coordinates which it crosses the x-axis and the turning points.

b) The graph of $y = f(ax)$ has a turning point at $(-8,6)$. Find the value of a.

c) The graph of $y = bf(x)$ has a turning point $(-4,2)$. Find the value of b.



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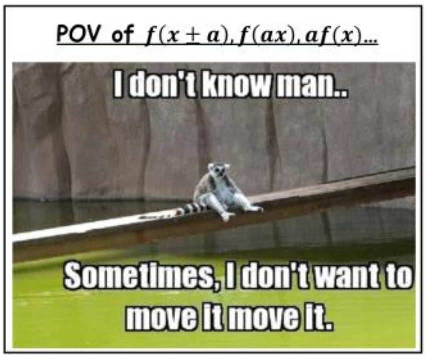
4.7 - Transforming Functions

Recap / Starter: Write down the geometrical transformation that is represented by each function:

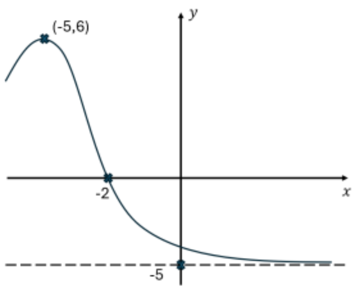
- $y = f(x + a)$
- $y = f(x) + a$
- $y = f(ax)$
- $y = af(x)$
- $y = f(-x)$
- $y = -f(x)$



Example

A dull man, Mr. Fan, made a sketch of a slide he saw in a park.
The curve of the slide can be modelled as $y = f(x)$.

a) Sketch the graph of $y = f(x + 1)$
b) Sketch the graph of $y = f(2x)$
c) Sketch $y - 1 = f(x)$
d) Sketch $5y = f(x) - 5$



Exam Practice (C1 May 2012 Q10)

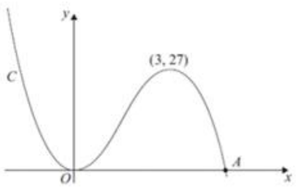


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = x^2(9 - 2x).$$

There is a minimum at the origin, a maximum at the point (3, 27) and C cuts the x-axis at the point A.

(a) Write down the coordinates of the point A. **(1)**

(b) On separate diagrams sketch the curve with equation

(i) $y = f(x + 3)$,

(ii) $y = f(3x)$.

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes. **(6)**

The curve with equation $y = f(x) + k$, where k is a constant, has a maximum point at (3, 10).

(c) Write down the value of k. **(1)**