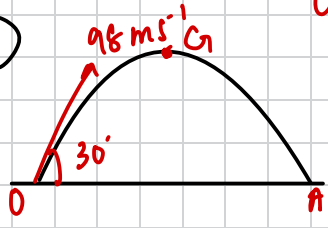


6.3: Projection at any angle

①



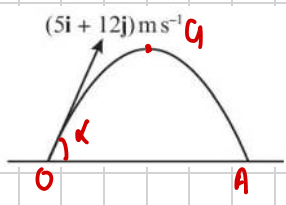
0 → G:

| | x | y |
|---|--------------------|-----------------------|
| s | s_x | s_y |
| u | $98 \cos 30^\circ$ | $98 \sin 30^\circ$ |
| v | | 0 → (greatest height) |
| a | 0 | -g |
| t | t | t |

⇒ y: $v = u + at \Rightarrow 0 = 98 \sin 30^\circ - gt \Rightarrow \frac{-98 \sin 30^\circ}{-g} = t$

$t = 5 \Rightarrow$ Total time $\Rightarrow 2t = 2(5) = 10$ seconds

②



0 → G:

| | x | y |
|---|-------|-----------------------|
| s | s_x | s_y |
| u | 5 | 12 |
| v | | 0 → (greatest height) |
| a | 0 | -g |
| t | t | t |

a) y: $v = u + at$

⇒ $0 = 12 - gt$

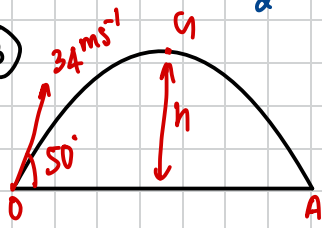
$t = \frac{12}{g} = 1.224489$

⇒ $2t = 2(1.22...)$

⇒ $t = 2.45$ seconds

b) x: $s = ut + \frac{1}{2}at^2 \Rightarrow s = 5(2.44...) + \frac{1}{2}(0)(t)^2 \Rightarrow 12.2 \text{ m (3sf)}$

③



0 → G:

| | x | y |
|---|--------------------|--------------------|
| s | s_x | h |
| u | $34 \cos 50^\circ$ | $34 \sin 50^\circ$ |
| v | | 0 |
| a | 0 | -g |
| t | t | t |

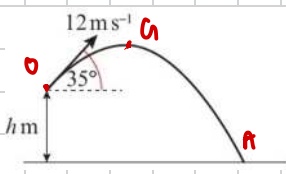
⇒ greatest height ($v = 0$)

⇒ y: $v^2 = u^2 + 2as$

⇒ $(0)^2 = (34 \sin 50^\circ)^2 + 2(-g)h$

⇒ $h = \frac{(34 \sin 50^\circ)^2}{2g} = 34.6 \text{ m (3sf)}$

④



0 → G:

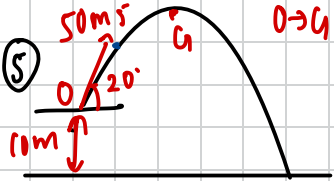
| | x | y |
|---|--------------------|--------------------|
| s | s_x | s_y |
| u | $12 \cos 35^\circ$ | $12 \sin 35^\circ$ |
| v | | |
| a | 0 | -g |
| t | 5 | 5 |

y: $s = ut + \frac{1}{2}at^2$

-s = $12 \sin(35^\circ)5 + \frac{1}{2}(-g)(5^2)$

∴ s = ∴ 88.085

S = 88.1 m



0 → G:

| | x | y |
|---|--------------------|--------------------|
| s | S_x | -10 |
| u | $50 \cos 20^\circ$ | $50 \sin 20^\circ$ |
| v | V_x | 0 |
| a | 0 | -g |
| t | t | t |

y: $S = ut + \frac{1}{2}at^2$

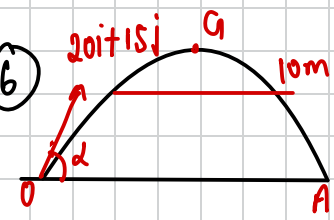
$$-10 = 50 \sin 20^\circ t - 4.9t^2$$

$$t = 4 \text{ seconds}$$

x: $S = ut + \frac{1}{2}at^2 \Rightarrow S = 4 \times 50 \cos 20^\circ$

$$\Rightarrow S = 187.9$$

$$\Rightarrow S = 188 \text{ m}$$



0 → G:

| | x | y |
|---|-------|----|
| s | S_x | 10 |
| u | 20 | 15 |
| v | | |
| a | 0 | -g |
| t | t | t |

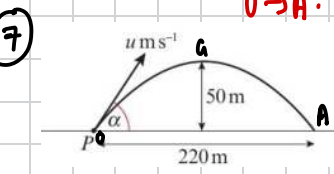
y: $S = ut + \frac{1}{2}at^2$

$$10 = 15t - 4.9t^2 \Rightarrow 4.9t^2 - 15t + 10 = 0$$

$$\Rightarrow t = 2.0801 \text{ or } t = 0.9811$$

$$\Rightarrow t = 2.0801 - 0.9811$$

$$\Rightarrow t = 1.099 = 1.10 \text{ sec (3sf)}$$



0 → A:

| | x | y |
|---|-------------------|-------------------|
| s | 220 | 50 |
| u | $u_x \cos \alpha$ | $u_y \sin \alpha$ |
| v | | |
| a | 0 | -g |
| t | t | t |

y: $v^2 = u^2 + 2as$

$$(0)^2 = u^2 \sin^2 \alpha + 2(-g)H$$

$$\Rightarrow 50 = \frac{u^2 \sin^2 \alpha}{2g} \quad \text{--- (1)}$$

→ Formula for horizontal projectile

x: $\text{Range} = \frac{u^2 \sin 2\alpha}{g}$

Alternative working out on last page

$$\Rightarrow 220 = \frac{u^2 \sin^2 \alpha}{9.8} \quad \text{--- (2)}$$

$$\Rightarrow \frac{(2)}{(1)} \Rightarrow \frac{220}{50} = \frac{u^2 \sin^2 \alpha}{\frac{u^2 \sin^2 \alpha}{2g}} \Rightarrow \frac{2 \sin \alpha}{\sin^2 \alpha} = \frac{4 \sin \alpha \cos \alpha}{\sin^2 \alpha}$$

$$\Rightarrow \frac{4 \cos \alpha}{\sin \alpha} \Rightarrow 4 \cot \alpha$$

$$\Rightarrow \frac{4}{\tan \alpha} = \frac{220}{50} \Rightarrow \tan \alpha = 4 \times \frac{50}{220}$$

$$\alpha = \arctan\left(\frac{20}{22}\right) = 42.27... \Rightarrow \alpha = 42.3^\circ \text{ (3sf)}$$

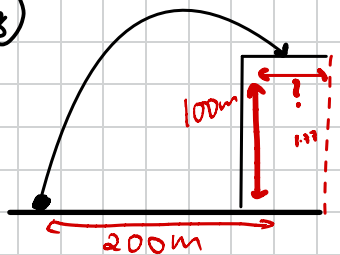
ii) $50 = \frac{u^2 \sin^2(42.27...)}{2g} \Rightarrow u^2 = 2165.8 \Rightarrow u = 46.5 \text{ ms}^{-1} \text{ (3sf)}$

b) $V_x = u_x \cos \alpha + at \Rightarrow V_x = u_x \cos \alpha = 34.435...$

$V_y = u_y \sin \alpha - g(5) \Rightarrow V_y = -17.695...$

$$\Rightarrow \text{speed} = \sqrt{V_x^2 + V_y^2} = \sqrt{34.435^2 + (-17.695)^2} = 38.7 \text{ ms}^{-1}$$

8



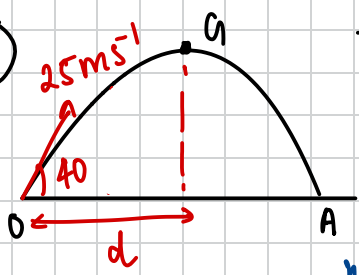
| | x | y |
|---|-------|-------|
| s | s_x | 100 |
| u | 36 | 48 |
| v | v_x | v_y |
| a | 0 | -g |
| t | t | t |

y: $s = ut + \frac{1}{2}at^2$
 $100 = 48t - 4.9t^2$
 $\Rightarrow 4.9t^2 - 48t + 100 = 0$
 $t = 6.7905 \quad t = 3.0053$

\Rightarrow distance from the edge of the cliff $\Rightarrow d - 200 = 44.5m$ (3sf)

x: $s = ut \Rightarrow d = u_x t = 6.7905 \times 36$
 $d = 244.4592379$

9



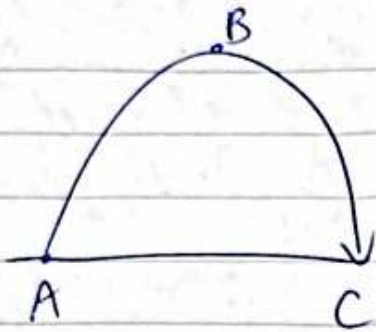
| | x | y |
|---|-------------------|-------------------|
| s | s_x | 4 |
| u | $25\cos 40^\circ$ | $25\sin 40^\circ$ |
| v | 0 | 0 |
| a | 0 | -g |
| t | t | t |

y: $s = ut + \frac{1}{2}at^2$
 $4 = 25\sin 40^\circ t - 4.9t^2$
 $\Rightarrow 4.9t^2 - 25\sin 40^\circ t + 4 = 0$
 $t = 3.008... \text{ or } t = 0.27137...$

x: $s = ut \Rightarrow d = 25\cos 40^\circ \times 3.008... = 57.6m$

BF MATHS

7a)



| A → B | | |
|-------|-----------------|-----------------|
| | x | y |
| s | | 50 |
| u | $u \cos \alpha$ | $u \sin \alpha$ |
| v | | 0 |
| a | | -9.8 |
| t | t | t |

| A → C | | |
|-------|-----------------|-----------------|
| | x | y |
| s | 220 | 0 |
| u | $u \cos \alpha$ | $u \sin \alpha$ |
| v | | |
| a | 0 | -9.8 |
| t | 2t | 2t |

$$y: v = u + at$$

$$0 = u \sin \alpha - 9.8t$$

$$t = \frac{u \sin \alpha}{9.8}$$

Also $v^2 = u^2 + 2as$

$$0 = (u \sin \alpha)^2 + 2(-9.8)(50)$$

$$u \sin \alpha = 14.5$$

$$u = \frac{14.5}{\sin \alpha}$$

$$x: s = ut + \frac{1}{2}at^2$$

$$220 = (u \cos \alpha)(2t)$$

$$110 = u t \cos \alpha$$

$$110 = u \left(\frac{u \sin \alpha}{9.8} \right) \cos \alpha$$

$$1078 = u^2 \sin \alpha \cos \alpha$$

$$1078 = \left(\frac{14.5}{\sin \alpha} \right)^2 \sin \alpha \cos \alpha$$

$$1078 = \frac{980 \sin \alpha \cos \alpha}{\sin^2 \alpha}$$

$$\frac{1078}{980} = \cot \alpha$$

$$\tan \alpha = \frac{980}{1078}$$

$$\alpha = \tan^{-1} \left(\frac{980}{1078} \right) = \underline{\underline{42.3^\circ}}$$

BF MATHS

$$u = \frac{14.5}{\sin 42.3^\circ} = \underline{\underline{46.5 \text{ ms}^{-1}}}$$