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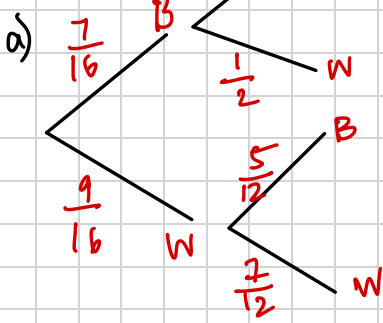
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Set : A

BRONZE:



$$b) P(B) = \left(\frac{7}{16} \times \frac{1}{2}\right) + \left(\frac{9}{16} \times \frac{5}{12}\right) = \frac{29}{64}$$

$$c) P(W(A) | W(B)) = \frac{P(W(A \cap B))}{P(W)(B)}$$

$$= \frac{\left(\frac{9}{16} \times \frac{7}{12}\right) + \left(\frac{7}{16} \times \frac{1}{2}\right)}{\frac{9}{16} + \frac{7}{32}} = \frac{3}{5}$$

SILVER:

Blue probability distribution:

$$\Rightarrow x + x + x + 3x + x + x = 1$$

$x$	1	2	3	4	5	6
$P(x=x)$	$x$	$x$	$x$	$3x$	$x$	$x$

$$x = \frac{1}{8} \Rightarrow P(x=4) = \frac{3}{8}$$

$$\Rightarrow P(\text{Blue} | 4) = \frac{P(\text{Blue} \cap 4)}{P(4)} \Rightarrow P(\text{Blue} \cap 4) = \frac{10}{72} \times \frac{3}{8} = \frac{10}{56} = \frac{5}{28}$$

$$P(A) = \frac{11 \frac{1}{6} + 10 \frac{3}{8}}{21} = \frac{67}{252} \Rightarrow P(\text{Blue} | 4) = \frac{5}{28} \div \frac{67}{252} = \frac{45}{67}$$

GOLD:

Let us take red counters as "x" ; Let us take blue counters as "3x"

$$\Rightarrow x + 3x = 4x$$

$$\Rightarrow \text{Probability of blue counters} = \frac{3x}{4x} \quad P(\text{Blue}) = \frac{3}{4}$$

$$\text{Probability of red counters} = \frac{x}{4x} \quad P(\text{Red}) = \frac{1}{4} \quad \frac{4-1}{15} = \frac{3}{15}$$

$$P(i) = P(x=1 | \text{red}) \times P(\text{Red}) + P(x=1 | \text{Blue}) \times P(\text{Blue}) \uparrow$$

$$P(i) = \left(\frac{1}{15}\right) \left(\frac{1}{4}\right) + \left(\frac{3}{15}\right) \left(\frac{3}{4}\right) = \frac{1}{60} + \frac{9}{60} = \frac{10}{60} = \frac{1}{6}$$

$$b) P(\text{Blue} | A) = \frac{27}{43}$$

$$P(\text{Blue} | A) = \frac{P(\text{Blue}=4) \times P(B)}{P(A)} \Rightarrow \frac{a \times \frac{3}{4}}{P(A)} = \frac{27}{43}$$

$$\Rightarrow \frac{3}{4}a = \frac{27}{43} \times P(A)$$

$$P(A) = P(X=4 | \text{red}) \times P(R) + P(Y=4 | \text{blue}) \times P(\text{blue})$$

$$P(A) = \left( \frac{4}{15} \times \frac{1}{4} \right) + \left( a \times \frac{3}{4} \right) = \frac{43a}{36}$$

$$\Rightarrow \frac{P(\text{Blue}=4) \times P(B)}{P(A)} = \frac{27}{43} \Rightarrow 27 \left( \frac{1}{15} + \frac{3a}{4} \right) = 43 \left( \frac{3a}{4} \right)$$

$$\Rightarrow \frac{27}{15} + \frac{81a}{4} = \frac{129}{4}a \Rightarrow \frac{27}{15} = \frac{48}{4}a \quad a = \frac{3}{20} = 0.15$$

$$\Rightarrow P(Y=1) = \frac{3}{15} = \frac{1}{5}$$

$$P(Y=2) = \frac{2}{15}$$

$$P(Y=3) = \frac{1}{15}$$

$$P(Y=4) = a = \frac{3}{20} = 0.15$$

$$P(Y=5) = b$$

$$\Rightarrow E(Y) = 1$$

$$\Rightarrow \frac{3}{15} + \frac{2}{15} + \frac{1}{15} + \frac{3}{20} + b = 1$$

$$\Rightarrow \frac{11}{20} + b = 1 \quad b = \frac{9}{20} = 0.45$$

# SET: B

## BRONZE:

$$P(Y) = 0.15 \quad P(Y \cap Z) = 0.05 \quad P(Y' \cap Z') = 0.75$$

a) i)  $P(Y \cup Z) = 1 - P(Y' \cap Z') = 1 - 0.75 = 0.25$

ii)  $P(Y \cup Z) = P(Y) + P(Z) - P(Y \cap Z) \Rightarrow 0.25 = 0.15 + P(Z) - 0.05$   
 $P(Z) = 0.15$

b)  $P(Y|Z) = \frac{P(Y \cap Z)}{P(Z)} = \frac{0.05}{0.15} = \frac{1}{3}$

c)  $P(Y \cap Z) \neq P(Y) \times P(Z) \Rightarrow 0.05 \neq (0.15) \times (0.15) = 0.0225$   
 So, the  $Y$  and  $Z$  are not statistically independent

d) If Zoe doesn't meet her target, Yvonne is more likely to miss hers and vice versa.

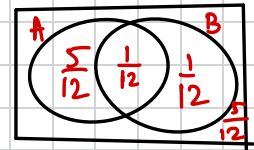
## SILVER:

$P(A) = 3P(B)$      $P(A \cup B) = \frac{7}{12}$     Let us take  $P(B)$  as  $x$ ;  $P(A) = 3x$

a)  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  ;  $P(A \cap B) = P(A) \times P(B) = 3x \times x = 3x^2$   
 $\frac{7}{12} = 3x + x - 3x^2 \Rightarrow -3x^2 + 4x - \frac{1}{12} = 0 \Rightarrow 36x^2 - 48x - 7 = 0$

$\Rightarrow x = \frac{1}{6}$  or  $x = \frac{1}{6}$

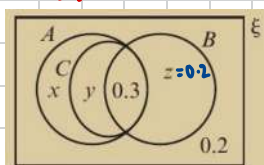
$x < 1$



b)  $P(A \cap B) = P(A) \times P(B) \Rightarrow 3x^2 = 3 \left(\frac{1}{6}\right)^2 = 3 \left(\frac{1}{12}\right) = \frac{1}{12}$

c)  $P(B|A') \Rightarrow \frac{P(B \cap A')}{P(A')} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$

## GOLD:



$P(A|B) = 0.6$   
 $\Rightarrow 0.6 = \frac{P(A \cap B)}{P(B)} \Rightarrow 0.6z = 0.18 \Rightarrow z = 0.2$   
 $\Rightarrow 0.6 = \frac{0.3}{z+0.3}$

$P(C|A) = 0.75$   
 $\Rightarrow 0.6z + 0.18 = 0.3$   
 $0.6z = 0.12$   
 $z = 0.2$

$0.75 = \frac{P(C \cap A)}{P(A)}$   
 $0.75 = \frac{y+0.3}{x+y+0.3}$

$$\Rightarrow 0.75x + 0.75y + 0.225 = y + 0.3$$

$$\Rightarrow 0.75x = 0.25y + 0.075$$

$$\Rightarrow 0.75(0.3 - y) = 0.25y + 0.075$$

$$0.225 - 0.75y = 0.25y + 0.075$$

$$0.15 = y$$

$$\Rightarrow x = 0.3 - y \Rightarrow 0.3 - 0.15 = 0.15$$

$$x + y + 0.3 + z + 0.2 = 1$$

$$x + y + 0.5 + 0.2 = 1$$

$$x + y = 0.3$$

$$x = 0.3 - y$$

$$z = 0.2 ; x = 0.15 ; y = 0.15$$

