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2·4: Probability formulae

① $P(A) = 0.5$; $P(B) = 0.3$; $P(A \cup B) = 0.6$

a) $P(A \cap B) \Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow 0.6 = 0.5 + 0.3 - P(A \cap B)$

$\Rightarrow -0.2 = -P(A \cap B) \Rightarrow P(A \cap B) = 0.2$

b) $P(A') = 1 - P(A) = 1 - 0.5 = 0.5$

c) $P(A' \cup B) = P(A') + P(B) - P(A' \cap B) \Rightarrow P(A' \cap B) = P(B) - P(A \cap B)$

$\Rightarrow 0.3 - 0.2 = 0.1$

$\Rightarrow 0.5 + 0.3 - 0.1 = 0.7$

d) $P(A \cup B') = P(A) + P(B') - P(A \cap B') \Rightarrow P(A \cap B') = P(A) - P(A \cap B)$

$\Rightarrow 0.5 + (1 - 0.3) - 0.3 \Rightarrow 0.5 + 0.7 - 0.3$ $\Rightarrow 0.5 - 0.2 = 0.3$

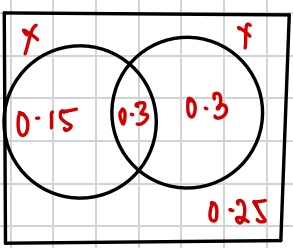
$\Rightarrow 0.9$

② $P(X) = 0.45$ $P(Y) = 0.6$ $P(X \cup Y) = 0.75$

a) $P(X \cap Y) \Rightarrow P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$\Rightarrow 0.75 = 0.45 + 0.6 - P(X \cap Y) \Rightarrow P(X \cap Y) = 0.3$

b)



i) $P(X' \cap Y') = 0.25$

ii) $P(Y|X) = \frac{0.3}{0.45} = \frac{2}{3}$

iii) $P(Y|X') = \frac{0.3}{0.55} = \frac{6}{11}$

c) $P(X \cap Y) = P(X) \times P(Y) \Rightarrow 0.3 \neq 0.45 \times 0.6 \Rightarrow 0.3 \neq 0.27$

So, the X and Y are not independent.

③ $P(A) = 0.4$ $P(B) = 0.7$ $P(A|B) = 0.5$

a) $P(A \cap B) \Rightarrow P(A|B) \times P(B) \Rightarrow 0.5 \times 0.7 = 0.35$

b) $P(B|A) = \frac{0.35}{0.4} = 0.875 = \frac{1}{8}$

c) $P(A \cup B) = 0.4 + 0.35 = 0.75$

④ $P(E) = x$ $P(F) = 1.5x$ $P(E \cap F) = 0.3$ $P(E' \cap F') = 0.05$

a) $P(E \cup F) = P(E) + P(F) - P(E \cap F) \Rightarrow x + 1.5x - 0.3 = 2.5x - 0.3$

b) $P(E \cup F) + P(E' \cup F') = 1 \Rightarrow 2.5x - 0.3 + 0.05 = 1 \Rightarrow 2.5x = 1.25$

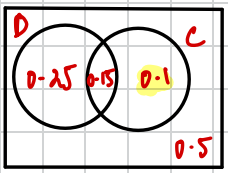
$x = 0.5$

c) i) $P(F) = 1.5x = 1.5(0.5) = 0.75$

ii) $P(E \cap F') = 0.5 - 0.3 = 0.2$

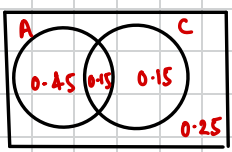
iii) $P(E|F') = \frac{0.2}{0.25} = \frac{4}{5} = 0.8$

⑤ Owned a dog = 0.4 ; Owned a cat = 0.25 ; either dog or cat or both (0.5)



$\Rightarrow P(C \cap D') = 0.1$

⑥ Abseiling (A) = 0.6 ; Canoeing (C) = 0.3



a) i) $P(C \cap A') = 0.15$

ii) $P(C' \cap A') = 0.25$

b) $0.25 \left[\frac{15}{60} = \frac{1}{4}\right]$

Let take there are 100 visitors

$(A) = 60 \quad (C) = 30 \Rightarrow \frac{50}{100} \times 30 = 15 \ (A \cap C)$

$(A' \cap C) = 15 \Rightarrow A' = 40 \Rightarrow C' = 70$

$P(A \cap C)' = 100 - (60 + 30 - 15) = 25$

⑦ $P(C|D) = 0.2 \quad P(C|D') = 0.6 \quad P(D) = 0.5$

a) $P(C \cap D) = 0.2 \times P(D) = 0.2 \times 0.5 = 0.1$

b) $P(C \cap D') = 0.6 \times P(D') = 0.6 \times 0.5 = 0.3$

c) $P(C) = 0.3 + 0.1 = 0.4$

d) $P(D|C) = \frac{0.1}{0.4} = \frac{1}{4} = 0.25$

e) $P(D'|C) = \frac{0.3}{0.4} = \frac{3}{4} = 0.75$

f) $P(D'|C') = \frac{0.2}{0.6} = \frac{1}{3}$

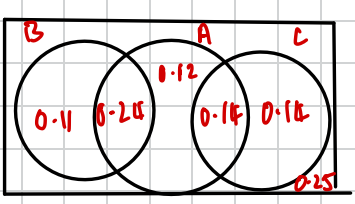
⑧ $P(A) = 0.5 \quad P(B) = 0.35 \quad P(A \cap B) = 0.24$

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow 0.5 + 0.35 - 0.24 = 0.61$

b) $P(A'|B) = \frac{0.35 - 0.24}{0.35} = \frac{0.11}{0.35} = \frac{11}{35}$

c) $P(A \cap C) = 0.14 \Rightarrow P(A) \times P(C) = 0.14 \Rightarrow P(C) = \frac{0.14}{0.5} = 0.28$

d)



e) $P((A \cup C)') = 0.14$

⑨ $P(A) = 0.5 \quad P(B) = 0.4 \quad P(C) = 0.2 \quad P(A \cap B) = 0.1$

a) $P(B \cap C) = P(B) \times P(C) = 0.4 \times 0.2 = 0.08$

b) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.5} = 0.2$

c) $P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{0.3}{0.4} = 0.75$

d) $P(B \cap C|A') = \frac{P(B \cap C \cap A')}{P(A')} = \frac{0.08}{0.5} = 0.16$

(10) a) $P(G \cup H) = 0.6 + 0.35 = 0.95$

b) $P(G \cap H') = 0.35$

c) $P(H|G) = \frac{1 - 0.75}{0.6} = \frac{0.25}{0.6} = \frac{5}{12}$

d) H and G are not independent, since $P(H|G) \neq P(H)$