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11.4 Using Integration

1) a) $a = 4t^2 - 3t^3$

$v = \int a\,dt$

$v = \int 4t^2 - 3t^3$

$v = \frac{4t^3}{3} - \frac{3t^4}{4} + c$

$c = 0$

$v = \frac{4t^3}{3} - \frac{3t^4}{4}$

b) $a = 5t + \frac{3t^2}{2}$

$v = \int a\,dt$

$v = \int 5t + \frac{3t^2}{2}$

$v = \frac{5t^2}{2} + \frac{t^3}{2} + c$

$c = 0$

$v = \frac{5t^2}{2} + \frac{t^3}{2}$

c) $a = \frac{2t^3 - t^2}{t}$

$v = \int a\,dt$

$v = \int \frac{2t^3 - t^2}{t}$ (t^2, t)

$v = \int 2t^2 - t$

$v = \frac{2t^3}{3} - \frac{t^2}{2} + c$

$c = 0$

$v = \frac{2t^3}{3} - \frac{t^2}{2}$

2) a) i) $v = 2t^3 - \frac{t}{3} + 1$

$s = \int v\,dt$

$s = \int 2t^3 - \frac{t}{3} + 1$

$s = \frac{t^4}{2} - \frac{t^2}{6} + t + c$

ii) $c = 0$

$s = \int_0^3 \frac{t^4}{2} - \frac{t^2}{6} + t$

$\left[\frac{t^4}{2} - \frac{t^2}{6} + t\right]_0^3 = 42 - 0 = 42$

$s = 42\text{m}$

b) i) $v = \frac{t}{3}(2t^2 + t^3)$

$s = \int v\,dt$

$s = \int \frac{2t^3}{3} + \frac{t^4}{3}$

$s = \frac{t^4}{6} + \frac{t^5}{15} + c$

ii) $c = 0$

$s = \int_0^3 \frac{t^4}{6} + \frac{t^5}{15}$

$\left[\frac{t^4}{6} + \frac{t^5}{15}\right]_0^3 = 29.7 - 0 = 29.7$

$s = 29.7\text{m}$

c) i) $v = 5t^2 + \frac{3\sqrt{t}}{2}$

$v = 5t^2 + \frac{3t^{1/2}}{2}$

$s = \int v\,dt$

$s = \int 5t^2 + \frac{3t^{1/2}}{2}$

$s = \frac{5t^3}{3} + t^{3/2} + c$

ii) $c = 0$

$s = \int_0^3 \frac{5t^3}{3} + t^{3/2}$

$s = \left[\frac{5t^3}{3} + t^{3/2}\right]_0^3 = 50.2 - 0 = 50.2$

$s = 50.2\text{m}$

3) $v = 10 + 3t - 2t^2$

$v = 0$

$0 = 10 + 3t - 2t^2$

$t = \frac{3+\sqrt{89}}{4}, \; t = \frac{3-\sqrt{89}}{4}$

$s = \int v\,dt$

$s = \int 10 + 3t - 2t^2\,dt$

$s = \int_0^2 10 + 3t - 2t^2$

$s = \left[10t + \frac{3t^2}{2} - \frac{2t^3}{3}\right]_0^2 = \frac{62}{3} - 0$

$= \frac{62}{3}$

$s = \frac{62}{3}\,m$

$x = 5 + \frac{62}{3} = \frac{77}{3}\,m$

4) $v = 4t^2 - 20t + 21$

$v = 0$

$0 = 4t^2 - 20t + 21$

$t = 7/2, \; t = 3/2$

$s = \int v\,dt$

$s = \int 4t^2 - 20t + 21$

$s = \frac{4t^3}{3} - 10t^2 + 21t$

$\int_{3/2}^{7/2} \frac{4t^3}{3} - 10t^2 + 21t = \frac{49}{6} - \frac{27}{2} = -\frac{16}{3}$

$\hookrightarrow 16/3$

$s = \frac{16}{3}\,m$

5) a) $a = t - 3\,ms^{-2}$

$v = \int a\,dt$

$v = \int t - 3$

$v = \frac{t^2}{2} - 3t + 4$

b) $\frac{t^2}{2} - 3t + 4 = 0$

$0.5t^2 - 3t + 4 = 0$

$t = 2, \; t = 4$

c) $s = \int v\,dt$

$s = \int \frac{t^2}{2} - 3t + 4$

$s = \int_2^4 \frac{t^3}{6} - \frac{3t^2}{2} + 4t$

$s = \left[\frac{t^3}{6} - \frac{3t^2}{2} + 4t\right]_2^4 = \frac{8}{3} - \frac{10}{3} = -\frac{2}{3}$

$\hookrightarrow \frac{2}{3}$

$s = \frac{2}{3}\,m$

6) $a = 5t + 9$

$v = \int a\,dt$

$v = \int 5t + 9$

$v = \frac{5t^2}{2} + 9t + C$

$v = 3$

$3 = \frac{5(0)^2}{2} + 9(0) + C$

$C = 3$

$\frac{5t^2}{2} + 9T + 3 = 7$

$\frac{5T^2}{2} + 9T + 3 - 7 = 0$

$\frac{5T^2}{2} + 9T - 4 = 0$

$2.5T^2 + 9T - 4 = 0$

$T = 0.4$ as $T \geq 0$

$T = -4$

7) $a = k(5 - t^2)$

$v = \int a\,dt$

$v = \int k(5 - t^2)\,dt$

$v = \int 5k - kt^2\,dt$

$v = 5kt - \frac{kt^3}{3} + C$

$v = 0 \Rightarrow C = 0$

$v = 5kt - \frac{kt^3}{3}$

Sub $v = 11$ and $t = 2$

$11 = 5k(2) - \frac{k(2^3)}{3}$

$11 = 10k - \frac{8k}{3}$

$11 = \frac{22k}{3}$

$k = \frac{3}{2}$

$v = 5(3/2)t - \frac{3/2\,t^3}{3}$

$v = \frac{15t}{2} - \frac{t^3}{2}$

$s = \int v\,t\,dt$

$s = \int \frac{15t}{2} - \frac{t^3}{2}\,dt$

$s = \frac{15t^2}{4} - \frac{t^4}{8} + C$

$s = 0 \Rightarrow C = 0$

$s = \left(\frac{15t^2}{4}\right) - \frac{t^4}{8}$ $\times 2$

$s = \frac{30t^2 - t^4}{8}$

$s = \frac{t^2(30 - t^2)}{8}$

8) $v = 5 + 8t^2 - 2t^3$

$a = \frac{dv}{dt}$

$a = 16t - 6t^2$

$0 = 2t(8 - 3t)$

$t = 0$ $t = \frac{8}{3}$

$\frac{d^2v}{dt^2} = 16 - 12t$

$16 - 12(8/3) = -16 \rightarrow -ve$

$\therefore T = \frac{8}{3}$

$$s = \int v\,dt$$

$$s = \int 5 + 8t^2 - 2t^3\,dt$$

$$s = 5t + \frac{8t^3}{3} - \frac{t^4}{2} + c$$

$c = 0$

$$s = 5t + \frac{8t^3}{3} - \frac{t^4}{2}$$

$$s = 5\left(\frac{8}{3}\right) + \frac{8(8/3)^3}{3} - \frac{(8/3)^4}{2}$$

$$s = \frac{3128}{81} \rightarrow 38.617$$

$s = 38.6\,m$