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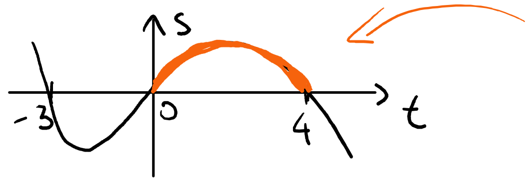
11.3 - Maxima and minima problems

If you need help on this chapter:
[A-Level Maths | Mechanics Year 1 | 11.3 - Maxima and minima Walkthrough | Edexcel]

Q1a) $s = 3t + 0.25t^2 - 0.25t^3$

$s = -t(0.25t^2 - 0.25t - 3)$ ← Factorising $-t$ to make the t^2 term positive for quadratic factorisation on the calculator

$s = -t(t-4)(t+3)$



$\boldsymbol{s \geq 0}$ **only when** $\boldsymbol{0 \leq t \leq 4}$

Q1b) Max distance: $\frac{ds}{dt} = 0$

$\frac{ds}{dt} = 3 + 0.5t - 0.75t^2 = 0$

$t = 2.36 \; or \; -1.69$

When $t = 2.36$, $s = 3(2.36) + 0.25(2.36)^2 - 0.25(2.36)^3$

$\boldsymbol{s = 5.19m \; (3s.f.)}$

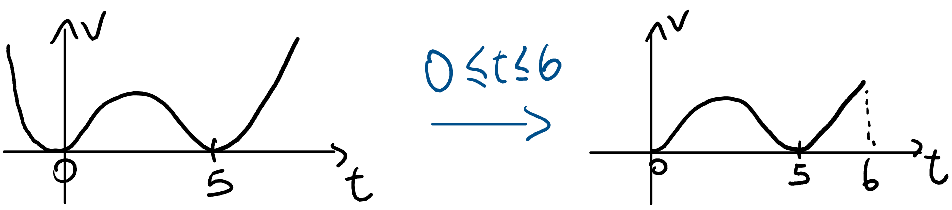
Q2a) $v = t^2(t-5)^2$ => Quartic graph

Watch this video if you need help on sketching quartic graphs:
[A-Level Maths | Pure Year 1 | 4.2 - Quartic graphs Walkthrough | Edexcel]

When $v = 0$, $0 = t^2(t-5)^2$

$t^2 = 0$ or $(t-5)^2 = 0$

$t = 0$ (double root), $t = 5$ (doube root)



Q2b) Max value of v : $\frac{dv}{dt} = 0$

$v = t^2(t-5)^2$

$= t^2(t^2 - 10t + 25)$

$= t^4 - 10t^3 + 25t^2$

Q2b continued

$$\frac{dv}{dt} = 4t^3 - 30t^2 + 50t = 0$$

$$t(4t^2 - 30t + 50) = 0$$

$t = 0$ or 2.5 or 5

v_{max} happens when $t = 2.5$

$$\boldsymbol{v_{max} = (2.5)^2(2.5 - 5)^2 = 39.0625ms^{-1}}$$

Q3a) $s = 0.5t^3 - 0.45t^2 - 2.1t + 6$

$$v = \frac{ds}{dt} = 1.5t^2 - 0.9t - 2.1$$

v_{min} means $\frac{dv}{dt} = 0$

$$\frac{dv}{dt} = 3t - 0.9 = 0$$

$$3t = 0.9$$

$\boldsymbol{t = \frac{0.9}{3} = 0.3}$ **second**

Q3b) When $t = 0.3$, $s = 0.5(0.3)^3 - 0.45(0.3)^2 - 2.1(0.3) + 6 = \mathbf{5.343m}$

Q3c) When $t = 0.3$, $v = 1.5(0.3)^2 - 0.9(0.3) - 2.1 = \mathbf{-2.235\ ms^{-1}}$

Q4a) $s = 5t^3 - t^4$

Return to starting point $\Rightarrow$ displacement $= 0 \Rightarrow s = 0$

When $s = 0$, $0 = 5t^3 - t^4$

$0 = t^3(5 - t) \Rightarrow t = 0$ or 5

$\therefore$ **Particle returns to starting point after 5 seconds.**

Q4b) $s = t^3(5 - t)$

When $0 \leq t \leq 5$, $t^3 \geq 0$ and $(5 - t) \geq 0$

Therefore $s \geq 0$ (non-negative)

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Q4c) s_{max} when $\frac{ds}{dt} = 0$

$$\frac{ds}{dt} = 15t^2 - 4t^3 = 0$$

$$t^2(15 - 4t) = 0$$

$$t = 0 \ \ or \frac{15}{4}$$

When $t = \frac{15}{4}$, $s = 5\left(\frac{15}{4}\right)^3 - \left(\frac{15}{4}\right)^4 =$ $\mathbf{65.9}\boldsymbol{m}$ **(3s.f.)**

Q5) $s = 0.5t^3 - 0.3t^2 - 0.8t + 6$

$$v = \frac{ds}{dt} = 1.5t^2 - 0.6t - 0.8$$

$$v_{min}\colon \frac{dv}{dt} = 0$$

$$\frac{dv}{dt} = 3t - 0.6 = 0$$

$$t = \frac{0.6}{3} = 0.2$$

When $t = 0.2$, $s = 0.5(0.2)^3 - 0.3(0.2)^2 - 0.8(0.2) + 6 = \mathbf{5.832}\ \boldsymbol{m}$

Q6a)

Method 1 (discriminant)

Never comes to rest ⇒ v never equals to 0

When $v = 0$, $3t^2 - 6t + 8 = 0$

$b^2 - 4ac = (-6)^2 - 4(3)(8) = -60, < 0$

∴ no real roots

∴ Particle never comes to rest

Method 2 (min. point)

$$v = 3t^2 - 6t + 8 = 0$$

$$\frac{dv}{dt} = 6t - 6$$

v_{min} at $\frac{dv}{dt} = 0 \Rightarrow t = 1$

When $t = 1$, $v = 3(1)^2 - 6(1) + 8 = 5$

Min. point $= (1, 5)$

The equation does not intersect with x-axis

∴ no real roots

∴ Particle never comes to rest

Q6b) $v = 3t^2 - 6t + 8$

$$\frac{dv}{dt} = 6t - 6$$

v_{min} at $\frac{dv}{dt} = 0 \Rightarrow 6t = 6 \Rightarrow t = 1$

When $t = 1$, $v_{min} = 3(1)^2 - 6(1) + 8 = \mathbf{5\ ms^{-1}}$

Q7) To verify if the car will ever reach the end of the track, *is Max Displacement > 5 km?*

$$s_{max} : \frac{ds}{dt} = 0$$

$$\frac{ds}{dt} = 4.8 + 2.4t - 0.03t^2 = 0$$

$t = 81.85$ or -1.95 (rej.)

When $t = 81.95$, $s = 4.8(81.95) + 1.2(81.95)^2 - 0.01(81.95)^3$

$s = 2948.7\ m = \mathbf{2.9487\ km}$, $< \mathbf{5km}$

∴ **It never reaches the end of track**. (How this question is worth 7 marks I have absolutely no idea!)

Q8) $v = at^2 + bt + c$

$$\frac{dv}{dt} = 2at + b$$

When $t = 3$, $\frac{dv}{dt} = 0$

$\Rightarrow 2a(3) + b = 0$

$6a + b = 0$ — ①

When $t = 0$, $v = 28$

$\Rightarrow 28 = a(0)^2 + b(0) + c \Rightarrow c = 28$

$\Rightarrow v = at^2 + bt + 28$

When $t = 3$, $v = 1$

$\Rightarrow 1 = a(3)^2 + b(3) + 28$

$9a + 3b = -27$ — ②

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Q8 continued

Solving equations ① and ② simultaneously:

$a = 3, \qquad b = -18$

$v = 3t^2 - 18t + 28$

Acceleration $= \frac{dv}{dt} = 6t - 18$

When $t = 4$,

$a = 6(4) - 18 = \mathbf{6\ ms^{-2}}$