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7.2 Finding Critical Values

1. Distribution $B(8, p)$

$H_0: p = 0.6$ $H_1: p < 0.6 \Rightarrow$ looking at bottom end

a) Assume H_0 is true then $X \sim B(8, 0.6)$

Using binomial distribution or calculator:

$$P(X = 0) = 0.0007 < 0.05$$

$$P(X \leq 1) = 0.0085 < 0.05$$

$$P(X \leq 2) = 0.0498 < 0.05$$

$$P(X \leq 3) = 0.1737 > 0.05$$

So critical region is:

$$0 \leq X \leq 2$$

b) Actual significance level is $P(X \leq 2) = 0.0498$

so actual error rate (when using critical region $X \leq 2$) is approx. 4.98%.

2. Distribution $B(15, p)$

$H_0: p = 0.3$ $H_1: p > 0.3 \Rightarrow$ looking at top end

a) Assume H_0 is true then $X \sim B(15, 0.3)$

$$P(X \geq 7) = 1 - P(X \leq 6)$$

$$= 1 - 0.8689$$

$$= 0.1311 > 0.1$$

$$P(X \geq 8) = 1 - P(X \leq 7)$$

$$= 1 - 0.9500$$

$$= 0.0500 < 0.1$$

So critical region is:

$$8 \leq X \leq 15$$

b) Actual significance level is $P(X \geq 8) = 0.0500$

so actual error rate when using critical region $X \geq 8$ is approx. 5.00%.

3. Distribution $B(25, p)$

$H_0: p = 0.42$ $H_1: p \neq 0.42 \Rightarrow$ looking at top and bottom end

(two-tailed test so significance level is 5% for each)

a) Assume H_0 is true then $X \sim B(25, 0.42)$

Bottom end

$$P(X=0) = 0.000001 < 0.05$$

$$P(X \leq 1) = 0.00002 < 0.05$$

$$P(X \leq 2) = 0.00021 < 0.05$$

$$P(X \leq 3) = 0.0013 < 0.05$$

$$P(X \leq 4) = 0.0055 < 0.05$$

$$P(X \leq 5) = 0.0184 < 0.05$$

$$P(X \leq 6) = 0.0495 < 0.05$$

$$P(X \leq 7) = 0.1106 > 0.05$$

Top end

$$P(X \geq 15) = 1 - P(X \leq 14)$$

$$= 1 - 0.9465$$

$$= 0.0535 > 0.05$$

$$P(X \geq 16) = 1 - P(X \leq 15)$$

$$= 1 - 0.9780$$

$$= 0.0220 < 0.05$$

$$P(X \geq 17) = 1 - P(X \leq 16)$$

$$= 1 - 0.9922$$

$$= 0.0078 < 0.05$$

So critical regions are:

$$0 \leq X \leq 6$$

and

$$16 \leq X \leq 25$$

b) Actual significance levels are $P(X \leq 6) = 0.0495$ and $P(X \geq 16) = 0.0220$

so actual error rate when using critical region $X \leq 6$ and $X \geq 16$ is approx. $4.95\% + 2.20\% = 7.15\%$

4.

- a) The test statistic is the number of plants in the sample that have blue flowers.

$$H_0: p = 0.2 \quad H_1: p < 0.2$$

- b) Assume H_0 is true then $X \sim B(30, 0.2)$

$$P(X=0) = 0.0012 < 0.05$$

$$P(X \leq 1) = 0.0105 < 0.05$$

$$P(X \leq 2) = 0.0442 < 0.05$$

$$P(X \leq 3) = 0.1227 > 0.05$$

So critical region is:

$$0 \leq X \leq 2$$

- c) Actual significance level is $P(X \leq 2) = 0.0442$

So probability of incorrectly rejecting the null hypothesis is 4.42%.

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5.

- a) The test statistic is the number of 5 star reviews in the sample.

$$H_0: p = 0.6 \quad H_1: p > 0.6$$

- b) Assume H_0 is true then $X \sim B(20, 0.6)$

$$\begin{aligned} P(X \geq 15) &= 1 - P(X \leq 14) \\ &= 1 - 0.8744 \\ &= 0.1256 > 0.1 \end{aligned}$$

$$\begin{aligned} P(X \geq 16) &= 1 - P(X \leq 15) \\ &= 1 - 0.9490 \\ &= 0.0510 < 0.1 \end{aligned}$$

$$\begin{aligned}
 P(X \geq 17) &= 1 - P(X \leq 16) \\
 &= 1 - 0.9840 \\
 &= 0.0160 < 0.1
 \end{aligned}$$

So critical region is:

$$16 \leq X \leq 20$$

c) Actual significance level is $P(X \geq 16) = 0.0510$

so actual error rate when using critical region $X \geq 16$ is approx. 5.10%.

6.

a) A suitable model would be $X \sim B(n, 0.05)$

$$H_0: p = 0.05 \quad H_1: p \neq 0.05$$

b) Assume H_0 is true then $X \sim B(50, 0.05)$

Bottom end	Top end
$P(X=0) = 0.0769 < 0.1$	$P(X \geq 5) = 1 - P(X \leq 4)$
$P(X \leq 1) = 0.2794 > 0.1$	$= 1 - 0.8964$
	$= 0.1036 > 0.1$
	$P(X \geq 6) = 1 - P(X \leq 5)$
	$= 1 - 0.9622$
	$= 0.0378 < 0.1$

So critical regions are:

$$0 \text{ and } 6 \leq X \leq 50$$

c) Actual significance levels are $P(X=0) = 0.0769$ and $P(X \geq 6) = 0.0378$

so actual error rate when using critical region $X=0$ and $X \geq 6$ is approx. $7.69\% + 3.78\% = 11.5\%$ (3 sf)

7. Distribution $B(25, p)$

$$H_0: p=0.3 \quad H_1: p \neq 0.3$$

a) Assume H_0 is true then $X \sim B(25, 0.3)$

Bottom end

$$P(X=0) = 0.0001 < 0.025$$

$$P(X \leq 1) = 0.0016 < 0.025$$

$$P(X \leq 2) = 0.0090 < 0.025$$

$$P(X \leq 3) = 0.0332 > 0.025 \Rightarrow \text{closest to } 0.025$$

$$P(X \leq 4) = 0.0905 > 0.025$$

Top end

$$P(X \geq 10) = 1 - P(X \leq 9)$$

$$= 1 - 0.8106$$

$$= 0.1894 > 0.025$$

$$P(X \geq 11) = 1 - P(X \leq 10)$$

$$= 1 - 0.9022$$

$$= 0.0978 > 0.025$$

$$P(X \geq 12) = 1 - P(X \leq 11)$$

$$= 1 - 0.9558$$

$$= 0.0442 > 0.025$$

$$P(X \geq 13) = 1 - P(X \leq 12)$$

$$= 1 - 0.9825$$

$$= 0.0175 < 0.025 \Rightarrow \text{closest to } 0.025$$

So critical regions are:

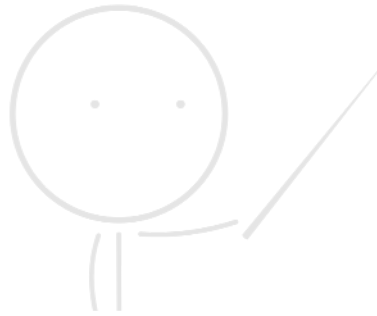
$$\{0 \leq X \leq 3\}$$

and

$$\{13 \leq X \leq 25\}$$

b) Actual significance levels are $P(X \leq 3) = 0.0332$ and $P(X \geq 13) = 0.0175$

so actual error rate when using critical region $X = 0$ and $X \geq 13$ is approx. $3.32\% + 1.75\% = 5.07\%$.



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