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Problem Solving - Set A

Bronze

Data in ascending order:

8 9 9 10 11 12 12 12 13 13 14

$$Q_1 = \frac{1}{4} \times 11 = 2.75\text{th} \rightarrow 9$$

$$Q_3 = \frac{3}{4} \times 11 = 8.25\text{th} \rightarrow 13$$

$$\begin{aligned} \text{IQR} &= 13 - 9 \\ &= 4 \end{aligned}$$

Silver

Number of hours	Frequency	Cumulative frequency
0-2	19	19
3-4	47	66
5-6	57	123
7-10	17	140

$$Q_1 = \frac{1}{4} \times 140 = 35\text{th value}$$

2.5	Q_1	4.5
19	35	66

$$\frac{Q_1 - 2.5}{4.5 - 2.5} = \frac{35 - 19}{66 - 19}$$

$$\frac{Q_1 - 2.5}{2} = \frac{16}{47}$$

$$Q_1 = \left(\left(\frac{16}{47} \right) \times 2 \right) + 2.5$$
$$= 3.18$$

$$Q_3 = \frac{3}{4} \times 140 = 105 \text{ th}$$

4.5	Q_3	6.5
66	105	123

$$\frac{Q_3 - 4.5}{6.5 - 4.5} = \frac{105 - 66}{123 - 66}$$

$$\frac{Q_3 - 4.5}{2} = \frac{39}{57}$$

$$Q_3 = \left(\left(\frac{39}{57} \right) \times 2 \right) + 4.5$$
$$= 5.87$$

$$IQR = 5.87 - 3.18$$
$$= 2.69 \text{ hours}$$

Gold

Score, x %	frequency	Cumulative Frequency
$0 \leq x < 20$	12	12
$20 \leq x < 40$	31	43
$40 \leq x < 60$	58	101
$60 \leq x < 80$	44	145
$80 \leq x < 100$	15	160

a) $P_{10} = 160 \times 0.10 = 16\text{th}$

$$\begin{array}{ccc} 20 & P_{10} & 40 \\ \hline 12 & 16 & 43 \end{array}$$

$$\frac{P_{10} - 20}{40 - 20} = \frac{16 - 12}{43 - 12}$$

$$\frac{P_{10} - 20}{20} = \frac{4}{31}$$

$$P_{10} = \left(\left(\frac{4}{31} \right) \times 20 \right) + 20$$

$$= 22.6$$

$P_{90} = 160 \times 0.90 = 144\text{th}$

$$\begin{array}{ccc} 60 & P_{90} & 80 \\ \hline 101 & 144 & 145 \end{array}$$

$$\frac{P_{90} - 60}{80 - 60} = \frac{144 - 101}{145 - 101}$$

$$\frac{P_{90} - 60}{20} = \frac{43}{44}$$

$$P_{90} = \left(\left(\frac{43}{44} \right) \times 20 \right) + 60$$
$$= 79.5$$

$$I PR = P_{90} - P_{10}$$
$$= 79.5 - 22.6$$
$$= 57.0$$

b) $\frac{175}{200} \times 100 = 87.5$

$$100 - 87.5 = 12.5$$

$$P_{12.5} = 160 \times 0.125 = 20$$

$\frac{20}{12}$	$\frac{P_{12.5}}{20}$	$\frac{40}{43}$
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$$\frac{P_{12.5} - 20}{40 - 20} = \frac{20 - 12}{43 - 12} \rightarrow \frac{P_{12.5} - 20}{20} = \frac{8}{31}$$

$$P_{12.5} = \left(\left(\frac{8}{31} \right) \times 20 \right) + 20$$

$$P_{12.5} = 25\%$$

- c) It is assumed that the scores are evenly distributed throughout the class, or that next year's scores will be similarly distributed to this year's scores.

Problem Solving - Set B

Bronze

Time, t (hours)	$2 \leq t < 3$	$3 \leq t < 4.5$	$4.5 \leq t < 5$	$5 \leq t < 7$	$7 \leq t < 10$
frequency	18	40	51	36	62

a) mean

midpoint \times frequency

$$2 \leq t < 3 \quad | \quad 2.5 \times 18 = 45$$

$$3 \leq t < 4.5 \quad | \quad 3.75 \times 40 = 150$$

$$4.5 \leq t < 5 \quad | \quad 4.75 \times 51 = 242.25$$

$$5 \leq t < 7 \quad | \quad 6 \times 36 = 216$$

$$7 \leq t < 10 \quad | \quad 8.5 \times 62 = 527$$

$$\frac{\sum \text{midpoint} \times \text{frequency}}{\sum \text{frequency}}$$

$$\frac{45 + 150 + 242.25 + 216 + 527}{18 + 40 + 51 + 36 + 62}$$

$$\frac{1180.25}{207}$$

$$= 5.70 \text{ hours}$$

Standard deviation:

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$\begin{aligned}\sum fx^2 &= (18 \times 2.5^2) + (40 \times 3.75^2) + (51 \times 4.75^2) + (36 \times 6^2) + (62 \times 8.5^2) \\ &= 112.5 + 562.5 + 1150.6875 + 1296 + 4479.5 \\ &= 7601.1875\end{aligned}$$

$$\sigma = \sqrt{\frac{7601.1875}{207} - \left(\frac{1180.25}{207}\right)^2}$$

$$= 2.05 \text{ hours}$$

- b) It is assumed all of the data points in each class are equal to the midpoint of the class interval

Silver

Mass, m (kg)	$3 \leq m < 3.5$	$3.5 \leq m < k$	$k \leq m < 4.0$	$4.0 \leq m < 4.5$	$4.5 \leq m < 6$
Frequency	17	21	33	54	15

a) midpoint \times frequency

$$3 \leq m < 3.5 \quad \left| \quad 3.25 \times 17 = 55.25\right.$$

$$3.5 \leq m < k \quad \left| \quad \frac{3.5+k}{2} \times 21\right.$$

$$k \leq m < 4.0 \quad \left| \quad \frac{k+4.0}{2} \times 33\right.$$

$$4.0 \leq m < 4.5 \quad \left| \quad 4.25 \times 54 = 229.5\right.$$

$$4.5 \leq m < 6 \quad \left| \quad 5.25 \times 15 = 78.75\right.$$

$$\frac{55.25 + \left(\frac{3.5+k}{2} \times 21\right) + \left(\frac{k+4.0}{2} \times 33\right) + 229.5 + 78.75}{140} = 4.09$$

140

$$\frac{55.25 + 10.5k + 16.5k + 66 + 229.5 + 78.75}{140} = 4.09$$

$$140$$

$$\frac{466.25 + 27k}{140} = 4.09$$

$$466.25 + 27k = 572.6$$

$$27k = 106.35$$

$$k = 3.94$$

$$b) \quad \sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$\begin{aligned}\sum fx^2 &= (17 \times (3.25)^2) + (21 \times (3.72)^2) + (33 \times (3.97)^2) + (54 \times (4.25)^2) + (15 \times (5.25)^2) \\ &= 2379.0911\end{aligned}$$

$$\sigma = \sqrt{\frac{2379.0911}{140} - (4.09)^2}$$

$$= 0.515 \text{ kg}$$

- c) It is assumed that all of the data points in each class are equal to the midpoint of the class interval

Gold

Salary, £'000	Frequency
$10 \leq x < 12$	2
$12 \leq x < 15$	5
$15 \leq x < 20$	12
$20 \leq x < 25$	13
$25 \leq x < 30$	7
$30 \leq x < 50$	8
$50 \leq x$	3

- The last class is open-ended
- The manager could assume a maximum value for the task, such as 100
- Use 100 as the upper class boundary of the last class:

Midpoint \times frequency

$10 \leq x < 12$	$11 \times 2 = 22$
$12 \leq x < 15$	$13.5 \times 5 = 67.5$
$15 \leq x < 20$	$17.5 \times 12 = 210$
$20 \leq x < 25$	$22.5 \times 13 = 292.5$
$25 \leq x < 30$	$27.5 \times 7 = 192.5$
$30 \leq x < 50$	$40 \times 8 = 320$
$50 \leq x < 100$	$75 \times 3 = 225$

$$\frac{22 + 67.5 + 210 + 292.5 + 192.5 + 320 + 225}{2 + 5 + 12 + 13 + 7 + 8 + 3}$$

$$\frac{1329.5}{50}$$

$$= 26.59 \times 1000$$

$$= \text{£}26590$$

- d) There could be a single very high value (such as the salary of the CEO) that makes the assumption in part b invalid.



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