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11.2 Integrating $f(ax+b)$

1a i] $(4x-5)^4$

$= \text{let } y = (4x-5)^4$

$\frac{dy}{dx} = 4 \times (4x-5)^3 \times 4$

$= 16(4x-5)^3$

ii] $(3-\cancel{8}2x)^6$

let $y = (3-2x)^6$

$\frac{dy}{dx} = 6(3-2x)^5 \times (-2)$

$= -12(3-2x)^5$

b i] $\int 16(4x-5)^3$

let $y = (4x-5)^{\cancel{3}4}$

$\frac{dy}{dx} = 16(4x-5)^3$

$\therefore \int 16(4x-5)^3 \, dx$

$= (4x-5)^3 + c$

ii] $\int (4x-5)^3 \, dx$

let $y = (4x-5)^4$

$\frac{dy}{dx} = 16(4x-5)^3$

$\therefore \int (4x-5)^3 \, dx$

$= \frac{1}{16}(4x-5)^3 + c$

iii] $\int -12(3-2x)^5 \, dx$

let $y = (3-2x)^6$

$\frac{dy}{dx} = -12(3-2x)^5$

11.2 Integrating $f(ax+b)$

1 b iii) Cont.

So $\int -12(3-2x)^5 \, dx$

$= (3-2x)^5 + c$

iv) $\int 2(3-2x)^5 \, dx$

Let $y = (3-2x)^6$

$\frac{dy}{dx} = -12(3-2x)^5$

$\therefore \int 2(3-2x)^5 \, dx$

$= \frac{-1}{6}(3-2x)^5 + c$

2a) $\int 6e^{3x}$

$= \frac{6}{3}e^{3x} + c$

$= 2e^{3x} + c$

b) $\int \cos(3x+2)$

$=$ Let $y = \sin(3x+2)$

$= \frac{dy}{dx} = 3\cos(3x+2)$

$\therefore \int \cos(3x+2)$

$= \frac{1}{3}\sin(3x+2) + c$

c) $(4x+1)^5$

$=$ Let $y = (4x+1)^6$

$= \frac{dy}{dx} = 6(4x+1)^5 \times 4$

$\frac{dy}{dx} = 24(4x+1)^5$

11.2 Integrating $f(ax+b)$

2c) Cont.

$$\therefore \int (4x+1)^5$$

$$= \frac{1}{24}(4x+1)^6 + c$$

d) $\operatorname{Cosec} 4x \operatorname{Cot} 4x$

$=$ let $y = -\operatorname{Cosec} 4x$

$$\frac{dy}{dx} = 4 \operatorname{Cosec} 4x \operatorname{Cot} 4x$$

$$\therefore \int \operatorname{Cosec} 4x \operatorname{Cot} 4x$$

$$= -\frac{1}{4} \operatorname{Cosec} 4x + c$$

e) $\operatorname{Sec}^2(8x-1)$

Let $y = \tan(8x-1)$

$$\frac{dy}{dx} = \operatorname{Sec}^2(8x-1) \times 8$$

$$= 8 \operatorname{Sec}^2(8x-1)$$

$$\therefore \int \operatorname{Sec}^2(8x-1)$$

$$= \frac{1}{8}\tan(8x-1) + c$$

f) $\operatorname{Cosec}^2(1-5x)$

Let $y = -\operatorname{Cot}(1-5x)$

$$\frac{dy}{dx} = \operatorname{Cosec}^2(1-5x) \times (-5)$$

$$= -5 \operatorname{Cosec}^2(1-5x)$$

$$\therefore \int \operatorname{Cosec}^2(1-5x)$$

$$= \frac{1}{5} \operatorname{Cot}(1-5x) + c$$

11.2 Integrating $f(ax+b)$

2g] $\frac{5}{2x-3}$

let $y = \ln|2x-3|$

$\frac{dy}{dx} = \frac{1}{2x-3} \times 2$

$= \frac{2}{2x-3}$

$\therefore \int \frac{5}{2x-3}$

$= \frac{5}{2} \ln|2x-3| + C$

h] $11e^{4-x}$

let $y = e^{4-x}$

$\frac{dy}{dx} = e^{4-x} \times (-1)$

$= -e^{4-x}$

So $\int 11e^{4-x}$

$= -11e^{4-x} + C$

3a] $\int \left(\frac{1}{(4x-3)^2} - (4x-3)^2\right) dx$

$= \int \left((4x-3)^{-2} - (4x-3)^2\right) dx$

let $y = (4x-3)^{-1}$

$\frac{dy}{dx} = -1(4x-3)^{-2} \times 4$

$= \frac{-4}{(4x-3)^2}$

let $y = (4x-3)^3$

$\frac{dy}{dx} = 3(4x-3)^2 \times 4$

$= 12(4x-3)^2$

So, $\int \left(\frac{1}{(4x-3)^2} - (4x-3)^2\right) dx$

$= \frac{-1}{4(4x-3)} - \frac{1}{12}(4x-3)^3 + C$

11.2 Integrating $f(ax+b)$

3b] $\int \sec^2 3x \left(\frac{1}{2}e^{5x}\cos^2 3x - 2\right) dx$

$= \int \frac{1}{2}\sec^2 3x \cos^2 3x e^{5x} - 2\sec^2 3x$

$= \int \frac{1}{2}e^{5x} - 2\sec^2 3x \quad dx$

let $y = e^{5x}$

$\frac{dy}{dx} = 5e^{5x}$

let $y = \tan 3x$

$\frac{dy}{dx} = 3\sec^2 3x$

So $\int \frac{1}{2}e^{5x} - 2\sec^2 3x$

$= \frac{1}{10}e^{5x} - \frac{2}{3}\tan 3x + c$

c] $\int \left(e^{2x} - \frac{1}{e^{2x}}\right)^2 dx$

$= \int \left(e^{4x} - 2 + \frac{1}{e^{4x}}\right) dx$

let $y = e^{4x}$

$\frac{dy}{dx} = 4e^{4x}$

let $y = e^{-4x}$

$\frac{dy}{dx} = -4e^{-4x}$

So $\int \left(e^{4x} - 2 + \frac{1}{e^{4x}}\right) dx$

$= \frac{1}{4}e^{4x} - 2x - \frac{1}{4}e^{-4x} + c$

d] $\int \frac{5 - 3\cos 2x}{\sin^2 2x} dx$

$= \int \frac{5}{\sin^2 2x} - 3\int \frac{\cos 2x}{\sin^2 2x}$

$= 5\int \operatorname{cosec}^2 2x - 3\int \operatorname{cosec} 2x \cot 2x$

let $y = -\cot 2x$

$\frac{dy}{dx} = 2\operatorname{cosec}^2 2x$

let $y = -\operatorname{cosec} 2x$

$\frac{dy}{dx} = 2\operatorname{cosec} 2x \cot 2x$

11.2 Integrating $f(ax+b)$

3d] Cont.

So ∴ $\int \frac{5 - 3\cos 2x}{\sin^2 2x} \, dx$

$= -\frac{5}{2} \cot 2x + \frac{3}{2} \operatorname{cosec} 2x + c$

4a] $\int_{-2}^{0} \frac{4}{6-4x} \, dx$

$\int 4(6-4x)^{-1}$

let $y = \ln|6 - 4x|$

$\frac{dy}{dx} = -\frac{4}{6-4x}$

$= \left[-\ln|6-4x|\right]_{-2}^{0}$

$= \left[-\ln|6-4(0)|\right] - \left[-\ln|6-4(-2)|\right]$

$= -\ln 6 + \ln 14$

$= \ln \frac{14}{6}$

$= \ln \frac{7}{3}$

5] $\int_{-\pi/8}^{\pi/8} \operatorname{cosec}^2 2x \, (1 - \cos 2x) \, dx$

$= \int \operatorname{cosec}^2 2x - \operatorname{cosec}^2 2x \cos 2x \, dx$

$= \int \operatorname{cosec}^2 2x - \operatorname{cosec} 2x \cot 2x \, dx$

let $y = -\cot 2x$

$\frac{dy}{dx} = 2\operatorname{cosec}^2 x$

let $y = -\operatorname{cosec} 2x$

$\frac{dy}{dx} = 2\operatorname{cosec} 2x \cot 2x$

$\int -\frac{1}{2}\cot 2x + \frac{1}{2}\operatorname{cosec} 2x$

11.2 Integrating $f(ax+b)$

4b) Cont.

$$\left[-\frac{1}{2}\cot 2x + \frac{1}{2}\operatorname{cosec} 2x\right]_{-\pi/8}^{\pi/8}$$

$$= \left[-\frac{1}{2}\cot 2\left(\frac{\pi}{8}\right) + \frac{1}{2}\operatorname{cosec} 2\left(\frac{\pi}{8}\right)\right] - \left[-\frac{1}{2}\left(\cot 2\left(-\frac{\pi}{8}\right)\right) + \frac{1}{2}\operatorname{cosec} 2\left(-\frac{\pi}{8}\right)\right]$$

$$= \left[-\frac{1}{2} + \frac{\sqrt{2}}{2}\right] - \left[\frac{1}{2} - \frac{\sqrt{2}}{2}\right]$$

$$= -\frac{1}{2} + \frac{\sqrt{2}}{2} - \frac{1}{2} + \frac{\sqrt{2}}{2}$$

$$= \sqrt{2} - 1$$

c) $\int_0^{\ln 4} (e^x - 4)^2 \, dx$

$$= \int e^{2x} - 8e^x + 16$$

let $y = e^{2x}$ | let $y = e^x$

$\frac{dy}{dx} = 2e^{2x}$ | $\frac{dy}{dx} = e^x$

$$\therefore \int e^{2x} - 8e^x + 16$$

$$= \frac{1}{2}e^{2x} - 8e^x + 16x$$

$$= \left[\frac{1}{2}e^{2x} - 8e^x + 16x\right]_0^{\ln 4}$$

$$= \left[\frac{1}{2}e^{2\ln 4} - 8e^{\ln 4} + 16\ln 4\right] - \left[\frac{1}{2}e^{2(0)} - 8e^0 + 16(0)\right]$$

$$= \left[\frac{1}{2}\cancel{e}^{\ln 4^2} - 8\cancel{e}^{\ln 4} + 16\ln 4\right] - \left[\frac{1}{2} - 8 + 0\right]$$

$$= \left[\frac{1}{2} \times 16 - 32 + 16\ln 4 - \frac{1}{2} + 8\right]$$

$$= 16\ln 4 - \frac{33}{2}$$

11.2 Integrating $f(ax+b)$

4d) $\int_{\frac{5\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{\sin^2(2\pi - x)}\,dx$

$= \int \operatorname{cosec}^2(2\pi - x)\,dx$

let $y = -\cot(2\pi - x)$

$\frac{dy}{dx} = -\operatorname{cosec}^2(2\pi - x)$

$\therefore \int \operatorname{cosec}^2(2\pi - x)$

$= \cot(2\pi - x)$

$= \left[\cot(2\pi - x)\right]_{\frac{5\pi}{8}}^{\frac{3\pi}{4}}$

$= \left[\cot\left(2\pi - \frac{3\pi}{4}\right)\right] - \left[\cot\left(2\pi - \frac{5\pi}{8}\right)\right]$

$= \left[\cot\frac{5\pi}{4} - \cot\frac{11\pi}{8}\right]$

$= 1 - \tan\left(\frac{\pi}{8}\right)$

5) $\int_1^a (2x+1)^2\,dx = 117$

$= \int (2x+1)^2\,dx$

let $y = (2x+1)^3$

$\frac{dy}{dx} = 3(2x+1)^2 \times 2 \implies 6(2x+1)^2$

So $\int (2x+1)^2 \implies \frac{1}{6}(2x+1)^3$

~~So~~ So $\int_1^a \left(\frac{1}{6}(2x+1)^3\right)$

$= \left[\frac{1}{6}(2x+1)^3\right]_1^a$

$= \left[\frac{1}{6}(2a+1)^3\right] - \left[\frac{1}{6}((2)(1)+1)^3\right] = 117$

5] Cont.

$\left[\frac{1}{6}(2a+1)^3\right] - \frac{9}{2} = 117$

$= \left[\frac{1}{6}(2a+1)^3\right] = \frac{243}{2}$

$= (2a+1)^3 = 729$

$2a+1 = \pm 9$

$2a = \pm 8$

$a = \pm 4$

Since 'a' is a positive constant,

$= a = 4$

6] $\int_1^4 \frac{k}{2x-1}\,dx = \ln 25$

$= \int \frac{k}{2x-1}\,dx$

let $y = k\ln|2x-1|$

So $\frac{dy}{dx} = \frac{k}{2x-1} \times 2$

$\frac{dy}{dx} = \frac{2k}{2x-1}$

So $\int \frac{k}{2x-1} \Longrightarrow \frac{1}{2}k\ln|2x-1|$

$= \left[\frac{1}{2}k\ln|2x-1|\right]_1^4 = \ln 25$

$= \left[\frac{1}{2}k\ln|2(4)-1|\right] - \left[\frac{1}{2}k\ln|2(1)-1|\right] = \ln 25$

$= \left[\frac{1}{2}k\ln|7|\right] - \left[\frac{1}{2}k\ln|1|\right] = \ln 25$

$= \frac{1}{2}k\ln 7 - \frac{1}{2}k\ln 1 = \ln 25$

$= k\ln 7 - k\ln 1 = 2\ln 25$

11.2 Integrating $f(ax+b)$

6) Cont.

$= k(\ln 7 - \ln 1) = 2\ln 25$

$= k\left(\ln \frac{7}{1}\right) = 2\ln 25$

$= k\ln 7 = 2\ln 25$

$= k = 2\left(\frac{\ln 25}{\ln 7}\right)$

$\Longrightarrow k = 2\left(\frac{\ln 25}{\ln 7}\right)$

7) $\int_{\ln 2}^{\ln 8} (e^x + 2)^2 \, dx$

(using RTSP)

$= \int (e^x + 2)^2 \, dx$

$= \int (e^{2x} + 4e^x + 4) \, dx$

$= \int e^{2x} \, dx + 4\int e^x \, dx + \int 4 \, dx$

$= \frac{1}{2}e^{2x} + 4e^x + 4x$

$= \left[\frac{1}{2}e^{2x} + 4e^x + 4x\right]_{\ln 2}^{\ln 8}$

$= \left[\frac{1}{2}e^{2\ln 8} + 4e^{\ln 8} + 4\ln 8\right] - \left[\frac{1}{2}e^{2\ln 2} + 4e^{\ln 2} + 4\ln 2\right]$

$= \frac{1}{2}e^{2\ln 8} + 4e^{\ln 8} + 4\ln 8 - \frac{1}{2}e^{2\ln 2} - 4e^{\ln 2} - 4\ln 2$

$= \frac{1}{2}\left[\cancel{e}^{\ln 8^2} - \cancel{e}^{\ln 2^2}\right] + 4\left[\cancel{e}^{\ln 8} - \cancel{e}^{\ln 2}\right] + 4\left[\ln 8 - \ln 2\right]$

$= \frac{1}{2}\left[8^2 - 2^2\right] + 4\left[8 - 2\right] + 4\left[\ln \frac{8}{2}\right]$

$= \frac{1}{2}[60] + 4[6] + 4\ln 4$

$= 30 + 24 + \ln 256$

$= 54 + \ln 256$

11.2 Integrating $f(ax+b)$

8] $\int_{\frac{\pi}{6k}}^{\frac{\pi}{3k}} \frac{\pi}{2}\cos kx \, dx = \frac{9\pi}{4}(1-\sqrt{3})$

$= \int \frac{\pi}{2}\cos kx$

$=$ let $y = \frac{\pi}{2}(\sin kx)$

$= \frac{dy}{dx} = k\frac{\pi}{2}\cos kx$

$=$ So $\int \frac{\pi}{2}\cos kx \Rightarrow \frac{\pi}{2k}\sin kx$

$= \left[\frac{\pi}{2k}\sin kx\right]_{\frac{\pi}{6k}}^{\frac{\pi}{3k}} = \frac{9\pi}{4}(1-\sqrt{3})$

$= \left[\frac{\pi}{2k}\sin\frac{\pi}{3\cancel{k}}(\cancel{k})\right] - \left[\frac{\pi}{2k}\sin\frac{\pi}{6\cancel{k}}(\cancel{k})\right] = \frac{9\pi}{4}(1-\sqrt{3})$

$= \frac{\pi}{2k}\left(\frac{\sqrt{3}}{2}\right) - \frac{\pi}{2k}\left(\frac{1}{2}\right) = \frac{9\pi}{4}(1-\sqrt{3})$

$= \frac{\sqrt{3}\pi - \pi}{\cancel{4}k} = \frac{9\pi}{\cancel{4}}(1-\sqrt{3})$

$= \frac{\pi(\sqrt{3}-1)}{k} = 9\pi(1-\sqrt{3})$

$= \cancel{\pi}(\sqrt{3}-1) = 9\cancel{\pi}k(1-\sqrt{3})$

$= \sqrt{3}-1 = 9k(1-\sqrt{3})$

$= \frac{1}{9}\left(\frac{\sqrt{3}-1}{1-\sqrt{3}}\right) = k$

$= \frac{1}{9} \times (-1) = k$

So $k = -\frac{1}{9}$