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Set A - Bronze

$y = e^{5x} - xe^{5x}$

When $y = 0$,

$0 = e^{5x} - xe^{5x}$

$0 = e^{5x}(1 - x)$

$\cancel{e^{5x} = 0}$ or $1 - x = 0$

$x = 1$

$\text{Area} = \int_0^1 e^{5x} - xe^{5x}\,dx$

$= \int e^{5x}\,dx - \int xe^{5x}\,dx$

IBP (ILATE)

$u = x \quad v' = e^{5x}$

$u' = 1 \quad v = \frac{1}{5}e^{5x}$

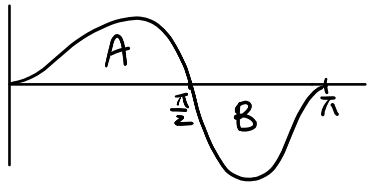
$= \left[\frac{1}{5}e^{5x} - \left(\frac{1}{5}xe^{5x} - \int \frac{1}{5}e^{5x} \times 1\,dx\right)\right]_0^1$

$= \left[\frac{1}{5}e^{5x} - \frac{1}{5}xe^{5x} + \frac{1}{25}e^{5x}\right]_0^1$

$= \left(\frac{1}{5}e^5 - \frac{1}{5}e^5 + \frac{1}{25}e^5\right) - \left(\frac{1}{5} - 0 + \frac{1}{25}\right)$

$= \frac{1}{25}e^5 - \frac{6}{25}$

Chapter 11 Problem Solving



Set A - Silver

Area $= A + B$

$$A = \int_0^{\frac{\pi}{2}} f(x)\,dx \quad , \quad B = \int_{\frac{\pi}{2}}^{\pi} f(x)\,dx$$

$$\int 2\sin x \cos x (1+\sin x)^4 \, dx$$

Let $u = 1 + \sin x \Rightarrow \frac{du}{dx} = \cos x$

$$\frac{du}{\cos x} = dx$$

x	0	$\frac{\pi}{2}$	π
u	1	2	1

$$\Rightarrow \int \ldots \, dx = \int 2(u-1)\cancel{\cos x}(u)^4 \times \frac{du}{\cancel{\cos x}}$$

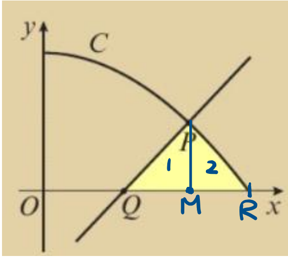
$$= \int 2u^5 - 2u^4 \, du$$

$$= \frac{u^6}{3} - \frac{2u^5}{5}$$

$$\text{Area } A = \left[\frac{u^6}{3} - \frac{2u^5}{5}\right]_1^2 = \left(\frac{2^6}{3} - \frac{2(2)^5}{5}\right) - \left(\frac{1}{3} - \frac{2}{5}\right) = \frac{43}{5}$$

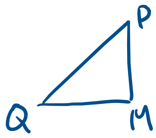
$$\text{Area } B = \left[\frac{u^6}{3} - \frac{2u^5}{5}\right]_2^1 = -\frac{43}{5}$$

$$\text{Total Area} = \frac{43}{5} + \frac{43}{5} = \underline{\underline{\frac{86}{5}}}$$



Set A - Gold

Total area = 1 + 2



$$\int_{\text{Point } P}^{\text{Point } R} \text{curve } C \, dx$$

When $t = \frac{\pi}{6}$, $x = 6\cos\left(\frac{\pi}{6}\right) = 3\sqrt{3}$
$y = 4\sin^2\left(\frac{\pi}{6}\right) = 1$
Coords of P: $(3\sqrt{3}, 1)$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{8\sin t\cos t}{-6\sin t} = -\frac{4}{3}\cos t$$

When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = -\frac{4}{3}\cos\left(\frac{\pi}{6}\right) = \frac{-2\sqrt{3}}{3}$

$\Rightarrow$ m of normal $= \frac{\sqrt{3}}{2}$

eq^{n} of normal at PQ: $y - y_1 = m(x - x_1)$

$$y - 1 = \frac{\sqrt{3}}{2}(x - 3\sqrt{3})$$

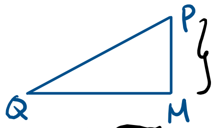
$$y = \frac{\sqrt{3}}{2}x - \frac{7}{2}$$

When $y = 0$, $0 = \frac{\sqrt{3}}{2}x - \frac{7}{2}$

$$x = \frac{7\sqrt{3}}{3}$$

$\Rightarrow Q\left(\frac{7\sqrt{3}}{3}, 0\right)$

Area 1:



1 $\quad = \frac{1}{2} \times \frac{2\sqrt{3}}{3} \times 1 = \frac{\sqrt{3}}{3}$

$3\sqrt{3} - \frac{7\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}$

$$\int y \frac{dx}{dt}\, dt = \int 4\sin^2 t \times (-6\sin t)\, dt$$
$$= \int -24\sin^3 t\, dt$$
$$= -24\int \sin^2 t \times \sin t\, dt$$
$$= -24\int (1-\cos^2 t)\sin t\, dt$$
$$= -24\int \sin t - \underline{\cos^2 t \sin t}\, dt$$

RCR
Let $y = \cos^3 t$
$y' = 3\cos^2 t(-\sin t)$
$y' = -3\cos^2 t \sin t$
$\int .. = \frac{1}{3}\cos^3 t$

$$= -24\left(-\cos t + \frac{1}{3}\cos^3 t\right)$$
$$= 24\cos t - 8\cos^3 t$$

Area ②: $\left[24\cos t - 8\cos^3 t\right]_{\frac{\pi}{6}}^{0}$ ← At point R, $t=0$ ← Point P

$$= \left[24\cos(0) - 8\cos^3(0)\right] - \left[24\cos\tfrac{\pi}{6} - 8\cos^3\left(\tfrac{\pi}{6}\right)\right]$$
$$= 16 - 9\sqrt{3}$$

Total area $= \frac{\sqrt{3}}{3} + 16 - 9\sqrt{3} = \frac{48 - 26\sqrt{3}}{3}$ (thank god!)

Set B - Bronze

a) $(x^2+x+2)\frac{dy}{dx} = 4xy+2y$

$(x^2+x+2)\frac{dy}{dx} = 2y(2x+1)$

$\int \frac{1}{2y}\,dy = \int \frac{2x+1}{x^2+x+2}\,dx$

RCR

Let $u = \ln|x^2+x+2|$

$u' = \frac{1}{x^2+x+2} \times (2x+1)$

$\int \text{''} = \ln|x^2+x+2|$

$\frac{1}{2}\ln y = \ln|x^2+x+2| + c$

$\ln y^{\frac{1}{2}} = \ln|x^2+x+2| + c$

$\cancel{e}^{\ln y^{\frac{1}{2}}} = e^{\ln|x^2+x+2|+c}$

$y^{\frac{1}{2}} = \cancel{e}^{\ln|x^2+x+2|} \times e^c$

Let $A = e^c$

$y^{\frac{1}{2}} = A(x^2+x+2)$

$y = A^2(x^2+x+2)^2$

b) When $x=0$, $y=1$

$1 = A^2(2)^2$

$A = \frac{1}{2}$

$\Rightarrow y = \frac{1}{4}(x^2+x+2)^2$

Set B - Silver

$$e^{2y-x}\frac{dy}{dx} = x(e^{2y}-1)$$

$$\frac{e^{2y}}{e^{x}}\frac{dy}{dx} = x(e^{2y}-1)$$

$$\int \frac{e^{2y}}{e^{2y}-1}\,dy = \int xe^{x}\,dx$$

RCR

Let $u = \ln|e^{2y}-1|$

$$u' = \frac{1}{e^{2y}-1}\times 2e^{2y}$$

$$\int \cdots = \frac{1}{2}\ln|e^{2y}-1|$$

IBP (ILATE)

$u = x \qquad v' = e^{x}$

$u' = 1 \qquad v = e^{x}$

$$\frac{1}{2}\ln|e^{2y}-1| = xe^{x} - \int e^{x}\,dx$$

$$\frac{1}{2}\ln|e^{2y}-1| = xe^{x} - e^{x} + c$$

$$\ln|e^{2y}-1| = 2xe^{x} - 2e^{x} + c$$

When $x=0$, $y=\ln 2$

$$\ln|e^{2\ln 2}-1| = 0 - 2 + c$$

$\ln(e^{\ln 4}-1)$

$= \ln(4-1)$

$= \ln 3$

$$\ln 3 = c - 2$$

$$c = 2 + \ln 3$$

$$\Rightarrow \underline{\underline{\ln|e^{2y}-1| = 2xe^{x} - 2e^{x} + 2 + \ln 3}}$$

Set B - Gold

$$\frac{dT}{dt} = \frac{1}{120000}T(80000-T)$$

$$\int \frac{1}{T(80000-T)}\,dT = \int \frac{1}{120000}\,dt$$

↓ Partial fractions

$$\boxed{\begin{aligned} \frac{1}{T(80000-T)} &\equiv \frac{A}{T} + \frac{B}{80000-T} \\ 1 &= A(80000-T) + BT \\ \text{When } T=0:\ & A = \frac{1}{80000} \\ \text{When } T=80000:\ & B = \frac{1}{80000} \end{aligned}}$$

$$\int \frac{1}{80000T} + \frac{1}{80000(80000-T)}\,dT = \frac{1}{120000}t + c$$

$$\frac{1}{80000}\int \frac{1}{T}\,dT + \frac{1}{80000}\int \frac{1}{80000-T}\,dT = \frac{1}{120000}t + c$$

$$\frac{1}{80000}\ln T + \frac{1}{80000}\times -\ln|80000-T| = \frac{1}{120000}t + c$$

$$\frac{1}{80000}\left(\ln T - \ln|80000-T|\right) = \frac{1}{120000}t + c$$

$$\ln\left(\frac{T}{80000-T}\right) = \frac{2}{3}t + c$$

$$\frac{T}{80000-T} = e^{\frac{2}{3}t + c}$$

$e^{\frac{2}{3}t} \times e^{c}$

Let $A = e^{c}$

$$\frac{T}{80000-T} = Ae^{\frac{2}{3}t}$$

When $t=0$, $T=16000$

$$\frac{1}{4} = A$$

$$\Rightarrow \frac{T}{80000-T} = \frac{1}{4}e^{\frac{2}{3}t}$$

$$\frac{T}{80000-T} = \frac{1}{4}e^{\frac{2}{3}t}$$

$$4T = e^{\frac{2}{3}t}(80000 - T)$$

$$4T = 80000e^{\frac{2}{3}t} - Te^{\frac{2}{3}t}$$

$$4T + Te^{\frac{2}{3}t} = 80000e^{\frac{2}{3}t}$$

factorise

$$T = \frac{80000e^{\frac{2}{3}t}}{4+e^{\frac{2}{3}t}}$$

divide top and bottom by $e^{\frac{2}{3}t}$

$$T = \frac{80000}{\frac{4}{e^{\frac{2}{3}t}} + 1}$$

$$T = \frac{80000}{1+4e^{-\frac{2}{3}t}}$$

(Thank god!)

b) When $t \to \infty$, $e^{-\frac{2}{3}t} \to 0$

$$\therefore T \to \frac{80000}{1+0} = 80000$$

($\because e^{-\frac{2}{3}t} = \frac{1}{e^{\frac{2}{3}t}}$

When $t \to \infty$, denominator $\to \infty$

$\frac{1}{\infty} \to 0$)