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11.3 - Using trigonometric identities

If you need help on this chapter: [A-Level Maths | Pure Year 2 | 11.3 - Integration with trig identities Walkthrough | Edexcel]

1a) $\int \sin^2 x \, dx$ $\rightarrow$ $\cos 2A \equiv 1 - 2\sin^2 A$

$2\sin^2 A \equiv 1 - \cos 2A$

$\sin^2 A \equiv \frac{1}{2} - \frac{1}{2}\cos 2A$

$= \int \frac{1}{2} - \frac{1}{2}\cos 2x \, dx$

$= \frac{1}{2}x - \frac{1}{4}\sin 2x + c$

1b) $\int 3\cot^2 x \, dx$ $\rightarrow$ $1 + \cot^2 A \equiv \operatorname{cosec}^2 A$

$\cot^2 A \equiv \operatorname{cosec}^2 A - 1$

$= 3\int \operatorname{cosec}^2 x - 1 \, dx$

use FB

$= 3(-\cot x - x) + c$

$= -3\cot x - 3x + c$

1c) $\int (1+\cos x)^2 \, dx$

$= \int 1 + 2\cos x + \cos^2 x \, dx$

$= x + 2\sin x + \int \cos^2 x \, dx$ $\longrightarrow$ $\cos 2A \equiv 2\cos^2 A - 1$

$\cos^2 A \equiv \frac{1}{2}\cos 2A + \frac{1}{2}$

$= x + 2\sin x + \int \frac{1}{2}\cos 2x + \frac{1}{2} \, dx$

$= x + 2\sin x + \frac{1}{4}\sin 2x + \frac{1}{2}x + c$

$= \frac{3x}{2} + 2\sin x + \frac{1}{4}\sin 2x + c$

1d) $\int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \tan^2 x \, dx$ $\rightarrow$ $\tan^2 A + 1 \equiv \sec^2 A$

$\tan^2 A \equiv \sec^2 A - 1$

$= \int \sec^2 x - 1 \, dx$

use FB

$= \tan x - x + c$

11.3 - Using trigonometric identities

2a) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\cos 2x}{\sin^2 x}\,dx = \int \frac{1-2\sin^2 x}{\sin^2 x}\,dx$

$$= \int \frac{1}{\sin^2 x} - \frac{2\cancel{\sin^2 x}}{\cancel{\sin^2 x}}\,dx$$

$$= \int \operatorname{cosec}^2 x - 2\,dx$$

FB

$$= \left[-\cot x - 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$= \left(-\cot\tfrac{\pi}{4} - 2\left(\tfrac{\pi}{4}\right)\right) - \left(-\cot\tfrac{\pi}{6} - 2\left(\tfrac{\pi}{6}\right)\right)$$

$$= -1 - \frac{\pi}{2} - \left(-\sqrt{3} - \frac{\pi}{3}\right)$$

$$= \underline{\underline{-1 + \sqrt{3} - \frac{\pi}{6}}}$$

2b) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sin x - \operatorname{cosec} x)^2\,dx$

$$= \int \sin^2 x - 2\sin x \operatorname{cosec} x + \operatorname{cosec}^2 x\,dx$$

$$= \int \frac{1}{2} - \frac{1}{2}\cos 2x - \frac{2\cancel{\sin x}}{\cancel{\sin x}} + \underline{\operatorname{cosec}^2 x}\,dx$$

FB

$$= \left[\frac{1}{2}x - \frac{1}{4}\sin 2x - 2x - \cot x\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \left[-\frac{3x}{2} - \frac{1}{4}\sin 2x - \cot x\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \left(-\frac{9\pi}{8} + \frac{1}{4} + 1\right) - \left(-\frac{3\pi}{8} - \frac{1}{4} - 1\right)$$

$$= \underline{\underline{-\frac{3\pi}{4} + \frac{5}{2}}}$$

11.3 - Using trigonometric identities

2c) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{(1-\cos x)^2}{\sin^2 x}\,dx = \int \frac{1-2\cos x+\cos^2 x}{\sin^2 x}\,dx$

$\displaystyle= \int \frac{1}{\sin^2 x} - \frac{2\cos x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x}\,dx$

$\displaystyle= \int \operatorname{cosec}^2 x - \frac{2\cos x}{\sin x}\times\frac{1}{\sin x} + \cot^2 x\,dx$

$\displaystyle= \int \underbrace{\operatorname{cosec}^2 x}_{\downarrow FB} - \underbrace{2\cot x\operatorname{cosec} x}_{\downarrow FB} + \cot^2 x\,dx$

$1+\cot^2 A \equiv \operatorname{cosec}^2 A$

$\cot^2 A \equiv \operatorname{cosec}^2 A - 1$

$\displaystyle= -\cot x + 2\operatorname{cosec} x + \int \underbrace{\operatorname{cosec}^2 x - 1}_{\downarrow FB}\,dx$

$\displaystyle= \left[-\cot x + 2\operatorname{cosec} x - \cot x - x\right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}}$

$\displaystyle= \left[-2\cot x + 2\operatorname{cosec} x - x\right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}}$

$\displaystyle= \left(\frac{2\sqrt{3}}{3} + \frac{4\sqrt{3}}{3} - \frac{2\pi}{3}\right) - \left(-\frac{2\sqrt{3}}{3} + \frac{4\sqrt{3}}{3} - \frac{\pi}{3}\right)$

$\displaystyle= \underline{\underline{\frac{4\sqrt{3}}{3} - \frac{\pi}{3}}}$

2d) $\displaystyle\int_0^{\frac{\pi}{4}} (\sec x - \tan x)^2\,dx = \int \sec^2 x - 2\sec x\tan x + \tan^2 x\,dx$

FB FB

$1+\tan^2 A \equiv \sec^2 A$

$\tan^2 A \equiv \sec^2 A - 1$

$\displaystyle= \tan x - 2\sec x + \int \sec^2 x - 1\,dx$

$\displaystyle= \left[\tan x - 2\sec x + \tan x - x\right]_0^{\frac{\pi}{4}}$

$\displaystyle= \left[2\tan x - 2\sec x - x\right]_0^{\frac{\pi}{4}}$

$\displaystyle= \left(2 - 2\sqrt{2} - \frac{\pi}{4}\right) - \left(0 - 2 - 0\right)$

$\displaystyle= \underline{\underline{4 - 2\sqrt{2} - \frac{\pi}{4}}}$

11.3 - Using trigonometric identities

3ai) $\cos(5x+2x) = \cos 5x \cos 2x - \sin 5x \sin 2x$

3aii) $\cos(5x-2x) = \cos 5x \cos 2x + \sin 5x \sin 2x$

3b) $\cos 7x + \cos 3x = \cos 5x \cos 2x - \cancel{\sin 5x \sin 2x} + \cos 5x \cos 2x + \cancel{\sin 5x \sin 2x}$

$= \underline{\underline{2\cos 5x \cos 2x}}$

3c) $\int 4\cos 5x \cos 2x \, dx$ Break 4 into 2×2 so we can use result in (b)

$= 2\int 2\cos 5x \cos 2x \, dx$

$= 2\int \cos 7x + \cos 3x \, dx$

$= 2\left(\frac{1}{7}\sin 7x + \frac{1}{3}\sin 3x\right) + c$

$= \underline{\underline{\frac{2}{7}\sin 7x + \frac{2}{3}\sin 3x + c}}$

4) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2 x \, dx \longrightarrow \cos 2A \equiv 2\cos^2 A - 1$

$2\cos^2 A \equiv \cos 2A + 1$

$= \int \frac{1}{2}\cos 2x + \frac{1}{2} \, dx \leftarrow \cos^2 A \equiv \frac{1}{2}\cos 2A + \frac{1}{2}$

$= \left[\frac{1}{4}\sin 2x + \frac{1}{2}x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= \left(0 + \frac{\pi}{4}\right) - \left(\frac{\sqrt{3}}{8} + \frac{\pi}{12}\right)$

$= \underline{\underline{\frac{-\sqrt{3}}{8} + \frac{\pi}{6}}}$

5) $\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \tan^2 2x \longrightarrow 1 + \tan^2 A \equiv \sec^2 A$

$\tan^2 A \equiv \sec^2 A - 1$

$= \int \sec^2 2x - 1 \, dx$

FB*

$= \left[\frac{1}{2}\tan 2x - x\right]_{\frac{\pi}{12}}^{\frac{\pi}{6}}$ * watch out for the 2x when integrating, need to ÷2

$= \left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right) - \left(\frac{\sqrt{3}}{6} - \frac{\pi}{12}\right)$

$= \underline{\underline{\frac{\sqrt{3}}{3} - \frac{\pi}{12}}}$

11.3 - Using trigonometric identities

6a) $\sin(6x+x) = \sin 6x \cos x - \sin x \cos 6x$

$\sin(6x-x) = \sin 6x \cos x + \sin x \cos 6x$

$\sin 7x + \sin 5x = \sin 6x \cos x - \cancel{\sin x \cos 6x} + \sin 6x \cos x + \cancel{\sin x \cos 6x}$

$= \underline{\underline{2\sin 6x \cos x}}$

6b) $\int 3 \sin 6x \cos x \, dx$

Make 3 into $\frac{3}{2} \times 2$ so I can use result in (a)

$= \frac{3}{2}\int 2 \sin 6x \cos 6x \, dx$

(a)

$= \frac{3}{2}\int \sin 7x + \sin 5x \, dx$

$= \frac{3}{2}\left[-\frac{1}{7}\cos 7x - \frac{1}{5}\cos 5x\right] + c$

$= \underline{\underline{-\frac{3}{14}\cos 7x - \frac{3}{10}\cos 5x + c}}$