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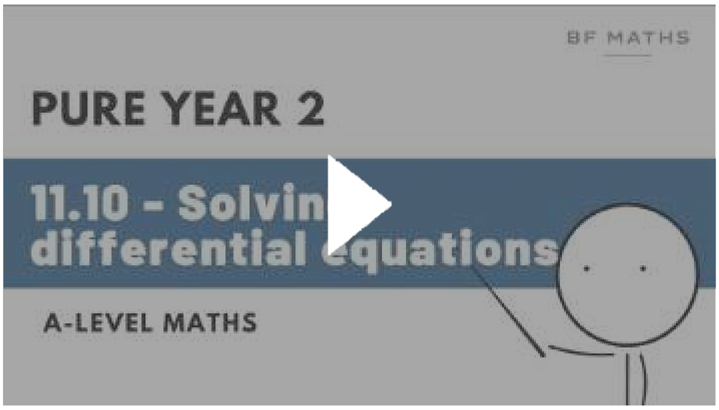
Thank you for using BF Maths for your maths revision!

If you need help on this chapter:
A-Level Maths | Pure Year 2 | 11.10 - Solving differential equations Walkthrough | Edexcel



11.10 - Solving differential equations

1a) $$\frac{1}{(x^2+6)^5} \times \frac{dy}{dx} = \frac{x}{y}$$

$$\int y\,dy = \int x(x^2+6)^5\,dx$$

1b) $$\int y\,dy = \int x(x^2+6)^5\,dx$$

RCR

$y = (x^2+6)^6$

$y' = 6(x^2+6)^5 \times 2x$

$y' = 12x(x^2+6)^5$

$\int \ldots dx = \frac{1}{12}(x^2+6)^6$

$$\frac{y^2}{2} = \frac{1}{12}(x^2+6)^6 + c$$

2a) $$\frac{dy}{dx} = e^{4x-3y}$$

$$\frac{dy}{dx} = e^{4x} \times e^{-3y}$$

$$\frac{dy}{dx} = \frac{e^{4x}}{e^{3y}}$$

$$\int e^{3y}\,dy = \int e^{4x}\,dx$$

$$\frac{1}{3}e^{3y} = \frac{1}{4}e^{4x} + c$$

$$e^{3y} = \frac{3}{4}e^{4x} + c$$

$$\ln(e^{3y}) = \ln\left(\frac{3}{4}e^{4x} + c\right)$$

$$3y = \ln\left(\frac{3}{4}e^{4x} + c\right)$$

$$y = \frac{1}{3}\ln\left(\frac{3}{4}e^{4x} + c\right)$$

I think the book answer is wrong.

11.10 - Solving differential equations

2b) $\frac{dy}{dx} = \sin^2 x \sec y$

$\int \frac{1}{\sec y}\,dy = \int \sin^2 x\,dx$

$\cos 2A = 1 - 2\sin^2 A$

$\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$

$\int \cos y\,dy = \int \frac{1}{2} - \frac{1}{2}\cos 2x\,dx$

$\sin y = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$

$y = \arcsin\left(\frac{1}{2}x - \frac{1}{4}\sin 2x + c\right)$

2c) $\frac{dy}{dx} = 6xy(x+1)$

$\int \frac{1}{y}\,dy = \int 6x(x+1)\,dx$

$\ln y = \int 6x^2 + 6x\,dx$

$\ln y = \frac{6x^3}{3} + \frac{6x^2}{2} + c$

$\ln y = 2x^3 + 3x^2 + c$

$y = e^{2x^3 + 3x^2 + c}$

$y = e^{2x^3 + 3x^2} \times e^c$

Let $A = e^c$

$y = Ae^{2x^3 + 3x^2}$

2d) $\frac{dy}{dx} = e^{2y} \sin x \cos^2 x$

$\int \frac{1}{e^{2y}}\,dy = \int \sin x \cos^2 x\,dx$

RCR

let $y = \cos^3 x$

$y' = 3\cos^2 x(-\sin x)$

$y' = -3\cos^2 x \sin x$

$\int \ldots dx = -\frac{1}{3}\cos^3 x$

$\int e^{-2y}\,dy = -\frac{1}{3}\cos^3 x + c$

$-\frac{1}{2}e^{-2y} = -\frac{1}{3}\cos^3 x + c$

$e^{-2y} = \frac{2}{3}\cos^3 x + c$

$-2y = \ln\left(\frac{2}{3}\cos^3 x + c\right)$

$y = -\frac{1}{2}\ln\left(\frac{2}{3}\cos^3 x + c\right)$

11.10 - Solving differential equations

3a) $(x+1)(x+2)\frac{dy}{dx} = xy + 3y$

$(x+1)(x+2)\frac{dy}{dx} = y(x+3)$

$\int \frac{1}{y}\,dy = \int \frac{x+3}{(x+1)(x+2)}\,dx$

Partial fractions

$\frac{x+3}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$x+3 = A(x+2) + B(x+1)$

$x=-2:\ 1 = B(-1) \Rightarrow B=-1$

$x=-1:\ 2 = A \Rightarrow A=2$

$\ln y = \int \frac{2}{x+1} - \frac{1}{x+2}\,dx$

$\ln y = 2\ln|x+1| - \ln|x+2| + c$

$\ln y = \ln|x+1|^2 - \ln|x+2| + c$

When $x=1,\ y=1$

$\ln 1 = \ln 4 - \ln 3 + c$

$0 = \ln\left(\frac{4}{3}\right) + c \Rightarrow c = -\ln\frac{4}{3}$ or $\ln\frac{3}{4}$ $\quad \ln\left(\frac{4}{3}\right)^{-1}$

$\ln y = \ln|x+1|^2 - \ln|x+2| + \ln\frac{3}{4}$

$\ln y = \ln\frac{(x+1)^2}{x+2} + \ln\frac{3}{4}$

$\ln y = \ln\left(\frac{3(x+1)^2}{4(x+2)}\right)$

$$\underline{\underline{y = \frac{3(x+1)^2}{4(x+2)}}}$$

3b) $\frac{dy}{dx} = \sin^2 y + e^x \sin^2 y$

$\frac{dy}{dx} = \sin^2 y(1+e^x)$

$\int \frac{1}{\sin^2 y} dy = \int (1+e^x) dx$

$\int \operatorname{cosec}^2 y \, dy = x + e^x + c$

$\downarrow$ use FB

$-\cot y = x + e^x + c$

$\cot y = -x - e^x + c$

When $x = \ln 2$, $y = \frac{\pi}{4}$

$\cot\left(\frac{\pi}{4}\right) = -\ln 2 - e^{\ln 2} + c$

$1 = \ln 2 - 2 + c$

$c = 3 - \ln 2$

$\cot y = -x - e^x + 3 - \ln 2$

$\underline{\underline{y = \operatorname{arccot}^{*}(-x - e^x + 3 - \ln 2)}}$

* arccot is very very uncommon in A-level maths.

3c) $\operatorname{cosec} y \cos^2 x \frac{dy}{dx} = \sin y$

$$\int \frac{\operatorname{cosec} y}{\sin y}\, dy = \int \frac{1}{\cos^2 x}\, dx$$

$$\int \operatorname{cosec}^2 y\, dy = \int \sec^2 x\, dx$$

FB FB

$$-\cot y = \tan x + c$$

When $x = \frac{\pi}{4}$, $y = \frac{\pi}{4}$

$$-\cot\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) + c$$

$$-1 = 1 + c \Rightarrow c = -2$$

$$-\cot y = \tan x - 2$$

$$\underline{\underline{\tan x + \cot y - 2 = 0}}$$

11.10 - Solving differential equations

3d) $\frac{2}{x} \times \frac{dy}{dx} = \frac{-2}{y}$

$$\int y\,dy = -\int x\,dx$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + c$$

When $x = 12$, $y = -5$

$$\frac{(-5)^2}{2} = \frac{-(12)^2}{2} + c \Rightarrow c = \frac{169}{2}$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + \frac{169}{2}$$

$$\underline{\underline{x^2 + y^2 = 169}}$$

4a) $\frac{dy}{dx} = 4x - 1$

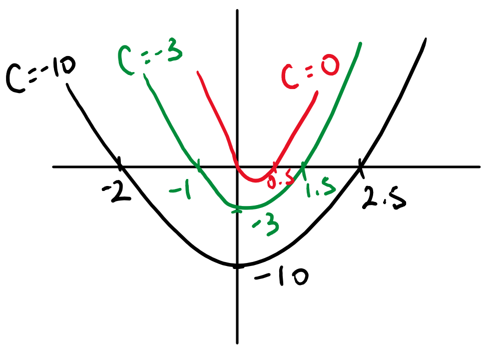
$$\int 1\,dy = \int (4x-1)\,dx$$

$$\underline{\underline{y = 2x^2 - x + c}}$$

4b) When $c = -10$, $y = 2x^2 - x - 10$ } roots $= 2.5, -2$

When $c = -3$, $y = 2x^2 - x - 3$ } roots $= 1.5, -1$

When $c = 0$, $y = 2x^2 - x$ } roots $= 0, 0.5$



11.10 - Solving differential equations

5) $xy\frac{dy}{dx} = (1+x^2)(1-y^2)$

$$\int \frac{y}{1-y^2}\,dy = \int \frac{1+x^2}{x}\,dx$$

RCR

let $z = \ln|1-y^2|$

$z' = \frac{1}{1-y^2} \times -2y$

$z' = \frac{-2y}{1-y^2}$

$\int \frac{1}{x} + \frac{x^2}{x}\,dx$

$= \int \frac{1}{x} + x\,dx$

$= \ln x + \frac{x^2}{2}$

$$-\frac{1}{2}\ln|1-y^2| = \ln x + \frac{x^2}{2} + c$$

$$\ln|1-y^2| = -2\ln x - x^2 + c$$

$$\ln|1-y^2| = -\ln(x^2) - x^2 + c$$

When $y=0,\ x=1$

$$\ln 1 = -\ln 1 - 1 + c \Rightarrow c = 1$$

$$\ln|1-y^2| = -\ln(x^2) - x^2 + 1$$

$$1-y^2 = e^{-\ln(x^2)-x^2+1}$$

$$1-y^2 = e^{\ln(x^{-2})} \times e^{-x^2+1}$$

$$1-y^2 = x^{-2}e^{-x^2+1}$$

$$1 - x^{-2}e^{x^2-1} = y^2$$

$$\underline{\underline{y^2 = 1-x^{-2}e^{-x^2+1}}}$$

6) $\sec y \frac{dy}{dx} = 2\cot y \sin^2 x + \cot y$

$\sec y \frac{dy}{dx} = \cot y (2\sin^2 x + 1)$

$\cot y = \frac{1}{\tan y}$

$\int \tan y \sec y \, dy = \int (2\sin^2 x + 1) \, dx$

FB

$\sec y = \int 2\sin^2 x \, dx + \int 1 \, dx$

$\cos 2A = 1 - 2\sin^2 A$
$2\sin^2 A = 1 - \cos 2A$

$\sec y = \int 1 - \cos 2x \, dx + x + c$

$\sec y = x - \frac{1}{2}\sin 2x + x + c$

$\sec y = 2x - \frac{1}{2}\sin 2x + c$

When $x = \frac{\pi}{4}$, $y = \frac{\pi}{4}$

$\sec\left(\frac{\pi}{4}\right) = \frac{2\pi}{4} - \frac{1}{2}\sin\left(\frac{2\pi}{4}\right) + c$

$\sqrt{2} = \frac{\pi}{2} - \frac{1}{2} + c \quad \Rightarrow \quad c = \frac{1}{2} + \sqrt{2} - \frac{\pi}{2}$

$$\sec y = 2x - \frac{1}{2}\sin 2x + \frac{1}{2} + \sqrt{2} - \frac{\pi}{2}$$

11.10 - Solving differential equations

7) $e^{2y-x}\frac{dy}{dx} = xe^{2y} - 4x$

$$\frac{e^{2y}}{e^x}\frac{dy}{dx} = x(e^{2y}-4)$$

$$\int \frac{e^{2y}}{e^{2y}-4}\,dy = \int xe^x\,dx$$

RCR

let $z = \ln|e^{2y}-4|$

$z' = \frac{1}{e^{2y}-4} \times 2e^{2y}$

$\int_{\prime} = \frac{1}{2}\ln|e^{2y}-4|$

IBP (ILATE)

$u = x \qquad v' = e^x$

$u' = 1 \qquad v = e^x$

$\int xe^x\,dx = xe^x - \int e^x\,dx$

$= xe^x - e^x + c$

$$\frac{1}{2}\ln|e^{2y}-4| = xe^x - e^x + c$$

$$\ln|e^{2y}-4| = 2xe^x - 2e^x + c$$

8a) $\frac{8x-11}{(2x-5)(x+2)} \equiv \frac{A}{2x-5} + \frac{B}{x+2}$

$$8x - 11 = A(x+2) + B(2x-5)$$

$x=-2$: $8(-2)-11 = B(-9)$

$B = 3$

$x = \frac{5}{2}$: $8\left(\frac{5}{2}\right) - 11 = A\left(\frac{9}{2}\right)$

$A = 2$

$$\frac{8x-11}{(2x-5)(x+2)} \equiv \frac{2}{2x-5} + \frac{3}{x+2}$$

11.10 - Solving differential equations

8b) $(2x-5)(x+2)\frac{dy}{dx} = y(8x-11)$

$$\int \frac{1}{y}\,dy = \int \frac{8x-11}{(2x-5)(x+2)}\,dx$$

$$\ln y = \int \frac{2}{2x-5} + \frac{3}{x+2}\,dx$$

$$\ln y = 2 \times \frac{1}{2}\ln|2x-5| + 3\ln|x+2| + c$$

$$\ln y = \ln\left[(2x-5)(x+2)^3\right] + c$$

$$y = e^{\ln\left[(2x-5)(x+2)^3\right] + c}$$

$$y = e^{\ln(\ldots)} \times e^c$$

Let $A = e^c$

$$\underline{\underline{y = A(2x-5)(x+2)^3}}$$

8c) When $x=3$, $y=25$

$$25 = A(1)(5)^3$$

$$A = \frac{25}{125} = \frac{1}{5}$$

$$\underline{\underline{y = \frac{1}{5}(2x-5)(x+2)^3}}$$