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9.4 The Product Rule

1a] $y = x(4-5x)^3$

$u = x$ $\qquad$ $v = (4-5x)^3$

$\frac{du}{dx} = 1$ $\qquad$ $\frac{dv}{dx} = 3(4-5x)^2(-5)$

$\frac{dv}{dx} = -15(4-5x)^2$

$$\frac{dy}{dx} = (4-5x)^3 - 15x(4-5x)^2$$

b] $y = 3x^3(2x-1)^6$

$u = 3x^3$ $\qquad$ $v = (2x-1)^6$

$\frac{du}{dx} = 9x^2$ $\qquad$ $\frac{dv}{dx} = 6(2x-1)^5 \times (2)$

$\frac{dv}{dx} = 12(2x-1)^5$

$$\frac{dy}{dx} = 9x^2(2x-1)^6 + 36x^3(2x-1)^5$$

c] $y = \frac{4}{x^2}(6x-7)^4$

$u = \frac{4}{x^2}$ $\qquad$ $v = (6x-7)^4$

$\frac{du}{dx} = \frac{-8}{x^3}$ $\qquad$ $\frac{dv}{dx} = 4(6x-7)^3(6)$

$\frac{dv}{dx} = 24(6x-7)^3$

$$\frac{dy}{dx} = \frac{96}{x^2}(6x-7)^3 + -\frac{8}{x^3}(6x-7)^4$$

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1d] $y = 8x^2 (2x^3 - 4)^{-3}$

$u = 8x^2$ $\qquad$ $v = (2x^3 - 4)^{-3}$

$\frac{du}{dx} = 16x$ $\qquad$ $\frac{dv}{dx} = -3(2x^3 - 4)^{-4} (6x^2)$

$\frac{dv}{dx} = -18x^2 (2x^3 - 4)^{-4}$

$\frac{dy}{dx} = 8x^2 \times -18x^2 (2x^3 - 4)^{-4} + 16x (2x^3 - 4)^{-3}$

$\frac{dy}{dx} = -144x^4 (2x^3 - 4)^{-4} + 16x (2x^3 - 4)^{-3}$

2a] $f(x) = x^2 \cos 2x$

$u = x^2$ $\qquad$ $v = \cos 2x$

$\frac{du}{dx} = 2x$ $\qquad$ $\frac{dv}{dx} = -2\sin 2x$

$\frac{dy}{dx} = -2x^2 \sin 2x + 2x \cos 2x$

b] $f(x) = e^{3x} \sin 5x$

$u = e^{3x}$ $\qquad$ $v = \sin 5x$

$\frac{du}{dx} = 3e^{3x}$ $\qquad$ $\frac{dv}{dx} = 5 \cos 5x$

$\frac{dy}{dx} = 5e^{3x} \cos 5x + 3e^{3x} (\sin 5x)$

$\frac{dy}{dx} = 5e^{3x} \cos 5x + 3e^{3x} \sin 5x$

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2c] $f(x) = e^{-x}(2x^2+3)^3$

$u = e^{-x}$ $\qquad$ $v = (2x^2+3)^3$

$\frac{du}{dx} = -e^{-x}$ $\qquad$ $\frac{dv}{dx} = 4x(2x^2+3)^2 \times 3$

$\frac{dv}{dx} = 12x(2x^2+3)^2$

$\frac{dy}{dx} = 12xe^{-x}(2x^2+3)^2 - e^{-x}(2x^2+3)^3$

d] $f(x) = -3\cos 4x \sin 2x$

$u = -3\cos 4x$ $\qquad$ $v = \sin 2x$

$\frac{du}{dx} = 12\sin 4x$ $\qquad$ $\frac{dv}{dx} = 2\cos 2x$

$\frac{dy}{dx} = -6\cos 4x \cos 2x + 12\sin 4x \sin 2x$

e] $f(x) = x^2 \ln x$

$u = x^2$ $\qquad$ $v = \ln x$

$\frac{du}{dx} = 2x$ $\qquad$ $\frac{dv}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = x + 2x\ln x$

f] $f(x) = e^{4x}\ln(\cos 3x)$

$u = e^{4x}$ $\qquad$ $v = \ln(\cos 3x)$

$\frac{du}{dx} = 4e^{4x}$ $\qquad$ $\frac{dv}{dx} = -\frac{3\sin 3x}{\cos 3x}$

$\frac{dv}{dx} = -3\tan 3x$

9.4 The Product Rule

2b] Cont.

$$\frac{dy}{dx} = -3e^{4x}\tan 3x + 4e^{4x}\ln(\cos 3x)$$

3] $y = 4x^3(1-2x^2)^4$

$u = 4x^3$ $\qquad$ $v = (1-2x^2)^4$

$\frac{du}{dx} = 12x^2$ $\qquad$ $\frac{dv}{dx} = 4(1-2x^2)^3(-4x)$

$\frac{dv}{dx} = -16x(1-2x^2)^3$

$$\frac{dy}{dx} = -64x^4(1-2x^2)^3 + 12x^2(1-2x^2)^4$$

$$\frac{dy}{dx} = -64(1)^4(1-2(1)^2)^3 + 12(1)^2(1-2(1)^2)^4$$

$$\frac{dy}{dx} = 64 + 12$$

$$\frac{dy}{dx} = 76$$

4a] $(x^4 - 3x^2)e^{-2x}$

$u = x^4 - 3x^2$ $\qquad$ $v = e^{-2x}$

$\frac{du}{dx} = 4x^3 - 6x$ $\qquad$ $\frac{dv}{dx} = -2e^{-2x}$

$$\frac{dy}{dx} = (4x^3-6x)e^{-2x} + (x^4-3x^2) - 2e^{-2x}$$

$$\frac{dy}{dx} = -2xe^{-2x}(x^3 - 2x^2 - 3x + 3)$$

9.4 The Product Rule

4b) $(4\cos 2x - \sin x)e^{3x}$

$u = 4\cos 2x - \sin x \qquad v = e^{3x}$

$\frac{du}{dx} = -8\sin 2x - \cos x \qquad \frac{dv}{dx} = 3e^{3x}$

$\frac{dy}{dx} = (-8\sin 2x - \cos x)e^{3x} + (4\cos 2x - \sin x)3e^{3x}$

$\frac{dy}{dx} = 12e^{3x}\cos 2x - 8e^{3x}\sin 2x - 3e^{3x}\sin x - e^{3x}\cos x$

c) $(x^2 - 5\cos \frac{1}{2}x)\ln 4x$

$u = x^2 - 5\cos \frac{1}{2}x \qquad v = \ln 4x$

$\frac{dv}{dx} = \frac{1}{x}$

$\frac{du}{dx} = 2x + \frac{5}{2}\sin \frac{1}{2}x$

$\frac{dy}{dx} = (x^2 - 5\cos \frac{1}{2}x)\left(\frac{1}{x}\right) + \left(2x + \frac{5}{2}\sin \frac{1}{2}x\right)(\ln 4x)$

$\frac{dy}{dx} = 2x\ln 4x + x - \frac{5}{x}\cos \frac{1}{2}x + \frac{5}{2}(\ln 4x)\sin \frac{1}{2}x$

5a) $e^{-x}\cos 3x$

$u = e^{-x} \qquad v = \cos 3x$

$\frac{du}{dx} = -e^{-x} \qquad \frac{dv}{dx} = -3\sin 3x$

$\frac{dy}{dx} = -e^{-x}\cos 3x - 3e^{-x}\sin 3x$

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5b] $x^4 \ln(\sin x)$

$u = x^4$ $\qquad$ $v = \ln(\sin x)$

$\frac{du}{dx} = 4x^3$ $\qquad$ $\frac{dv}{dx} = \frac{\cos x}{\sin x}$

$\frac{dv}{dx} = \cot x$

$\frac{dy}{dx} = (\cot x)x^4 + 4x^3 \ln(\sin x)$

6a] $y = 2x^4(2-5x^2)^3$

$u = 2x^4$ $\qquad$ $v = (2-5x^2)^3$

$\frac{du}{dx} = 8x^3$ $\qquad$ $\frac{dv}{dx} = -30x(2-5x^2)^2$

$\frac{dy}{dx} = 8x^3(2-5x^2)^3 - 60x^5(2-5x^2)^2$

$\frac{dy}{dx} = x^3(2-5x^2)^2(8(2-5x^2) - 60x^2)$

$\frac{dy}{dx} = x^3(2-5x^2)^2(16 - 40x^2 - 60x^2)$

$\frac{dy}{dx} = x^3(2-5x^2)^2(16 - 100x^2)$

$n = 2$, $a = -100$, $b = 0$, $c = 16$

b] Stationary points is when $\frac{dy}{dx} = 0$

$x^3(2-5x^2)(16-100x^2) = 0$

$x = 0$ $\qquad$ $x^2 = \frac{2}{5}$ $\qquad$ $x = \pm \frac{\sqrt{2}}{\sqrt{5}}$

9.4 The Product Rule

6b] Cont.

So $x = 0$, $x = \pm\sqrt{\frac{2}{5}}$, $x = \pm\frac{2}{5}$

7a] $f(x) = e^{4x}\sin x$

$u = e^{4x}$ $\quad v = \sin x$

$\frac{du}{dx} = 4e^{4x}$ $\quad \frac{dv}{dx} = \cos x$

$\frac{dy}{dx} = e^{4x}\cos x + 4e^{4x}\sin x$

$\frac{dy}{dx} = e^{4x}(4\sin x + \cos x) = 0$

let $\frac{dy}{dx} = 0$

$= e^{4x}(4\sin x + \cos x) = 0$

$= 4\sin x = -\cos x$

$= -\tan x = \frac{1}{4}$

$= \tan x = -\frac{1}{4}$

Hence proved.

b] $x = \frac{\pi}{2}$

$f(x) = e^{4(\frac{\pi}{2})}\sin\frac{\pi}{2}$

$f(x) = e^{2\pi}$

$f'(x) = e^{2\pi}\cos\frac{\pi}{2} + 4e^{2\pi}\sin\frac{\pi}{2}$

9.4 The Product Rule

7b] Cont.

$$f'(x) = 4e^{2x}$$

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - e^{2\pi} = \frac{-1}{4e^{2\pi}}\left(x - \frac{\pi}{2}\right)$$

$$y = \frac{-x}{4e^{2\pi}} + \frac{\pi}{8e^{2\pi}} + e^{2\pi} \qquad (\times 4e^{2\pi})$$

$$4e^{2\pi}y = -x + \frac{\pi}{2} + 4e^{4\pi}$$

$$= x + 4e^{2\pi}y - 4e^{4\pi} - \frac{\pi}{2} = 0$$

8] $y = (x - \pi)\sin 2x \qquad x = \frac{3\pi}{4}$

$u = (x - \pi) \qquad v = \sin 2x$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = 2\cos 2x$

$$\frac{dy}{dx} = 2(x - \pi)\cos 2x + \sin 2x$$

$$\frac{dy}{dx} = 2\left(\frac{3\pi}{4} - \pi\right)\cos 2\left(\frac{3\pi}{4}\right) + \sin 2\left(\frac{3\pi}{4}\right)$$

$$\frac{dy}{dx} = -1$$

$$y = \left(\frac{3\pi}{4} - \pi\right)\sin 2\left(\frac{3\pi}{4}\right)$$

9.4 The Product Rule

8] Cont.

$$y = -\frac{\pi}{4} \times (-1)$$

$$y = \frac{\pi}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{\pi}{4} = -1\left(x - \frac{3\pi}{4}\right)$$

$$y = -x + \frac{3\pi}{4} + \frac{\pi}{4}$$

$$y = -x + \pi$$

$$x + y = \pi$$

Hence proved.