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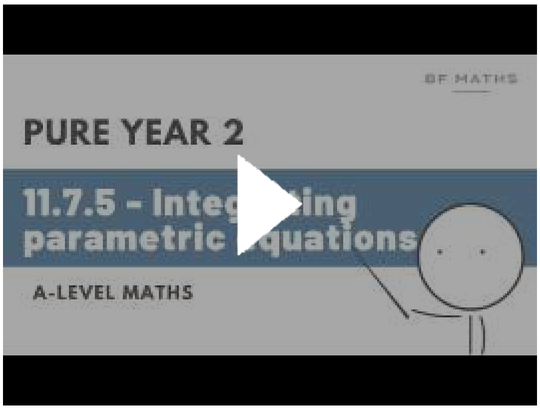
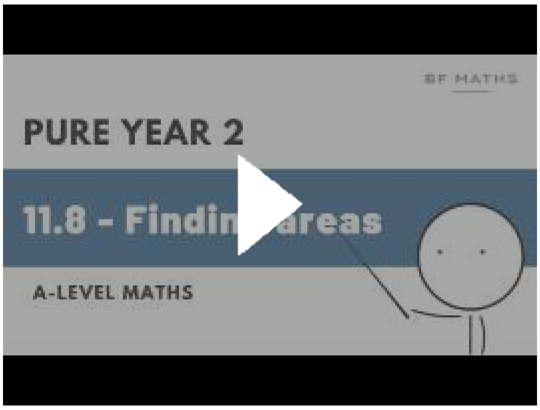
Thank you for using BF Maths for your maths revision!

If you need help on this chapter:

[A-Level Maths | Pure Year 2 | 11.7.5 - Integrating parametric equations Walkthrough | Edexcel]

[A-Level Maths | Pure Year 2 | 11.8 - Finding area Walkthrough | Edexcel]





11.8 - Finding areas

1) $\int_0^2 \frac{4}{(1+2x)^2}\,dx = 4\int_0^2 (1+2x)^{-2}\,dx$

RCR
$y=(1+2x)^{-1}$
$y' = -1(1+2x)(2)$
$\int \ldots = -\frac{1}{2}(1+2x)^{-1}$

$= 4\left[-\frac{1}{2}(1+2x)^{-1}\right]_0^2$

$= -2\left[(1+2x)^{-1}\right]_0^2$

$= -2\left[5^{-1} - 1^{-1}\right]$

$= \frac{8}{5}$

2a) $y = x\ln\left(\frac{1}{2}x\right)$

when $y=0$,

$0 = x\ln\left(\frac{1}{2}x\right)$

$x=0$ or $\ln\left(\frac{1}{2}x\right)=0$

$\frac{1}{2}x = e^0$

$\frac{1}{2}x = 1$

$x = 2$

$\therefore$ $A(2,0)$

2b) $R_1 = \int_1^2 x\ln\left(\frac{1}{2}x\right)dx$

IBP (ILATE)

$u = \ln\left(\frac{1}{2}x\right)$ $\quad v' = x$

$u' = \frac{1}{\frac{1}{2}x}\times\frac{1}{2}$ $\quad v = \frac{x^2}{2}$

$u' = \frac{1}{x}$

$= \left[\frac{x^2}{2}\ln\left(\frac{1}{2}x\right) - \int \frac{x^2}{2}\times\frac{1}{x}\,dx\right]_1^2$

$= \left[\frac{x^2}{2}\ln\left(\frac{1}{2}x\right) - \int \frac{x}{2}\,dx\right]_1^2$

$= \left[\frac{x^2}{2}\ln\left(\frac{1}{2}x\right) - \frac{x^2}{4}\right]_1^2$

$= \left[\frac{2^2}{2}\ln\left(\frac{1}{2}\times 2\right) - \frac{2^2}{4}\right] - \left[\frac{1}{2}\ln\left(\frac{1}{2}\right) - \frac{1}{4}\right]$

$= -1 - \frac{1}{2}\ln\left(\frac{1}{2}\right) + \frac{1}{4} = -\frac{3}{4} - \ln\left(\frac{1}{2}\right)$

R_1 is underneath x-axis so change the sign for the area.

$\therefore$ Area $= \frac{3}{4} + \ln\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \ln\left(2^{-1}\right)$

$= \frac{3}{4} - \ln 2$

11.8 - Finding areas

2c) $R_2 = \int_2^3 x\ln\left(\frac{1}{2}x\right)dx$

$= \left[\frac{x^2}{2}\ln\left(\frac{1}{2}x\right) - \frac{x^2}{4}\right]_2^3$

$= \left(\frac{3^2}{2}\ln\left(\frac{3}{2}\right) - \frac{3^2}{4}\right) - \left(\frac{2^2}{2}\ln(1) - \frac{2^2}{4}\right)$

$= \frac{9}{2}\ln\left(\frac{3}{2}\right) - \frac{9}{4} + 1$

$= \frac{9}{2}\ln\left(\frac{3}{2}\right) - \frac{5}{4}$

$\therefore$ Total area $= R_1 + R_2$

$= \frac{3}{4} - \ln 2 + \frac{9}{2}\ln\left(\frac{3}{2}\right) - \frac{5}{4}$

$= \underline{\underline{\frac{9}{2}\ln\left(\frac{3}{2}\right) - \ln 2 - \frac{1}{2}}}$

3a) P = intersection point

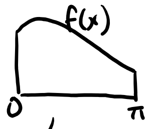
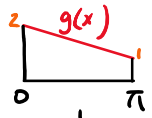
$2\cos\frac{1}{2}x + 1 = -\frac{1}{\pi}x + 2$

I don't think this equation is solvable using A-level methods.

If we let $x = \pi$

$\left.\begin{aligned} 2\cos\left(\tfrac{1}{2}\pi\right) + 1 &= 1 \\ -\tfrac{1}{\pi}(\pi) + 2 &= 1 \end{aligned}\right\}$ so when $x = \pi$, $y = 1$

$\underline{\underline{P = (\pi, 1)}}$

3b) $R =$  $-$ 

$\int_0^\pi f(x)\,dx$

Trapezium
When $x = 0$,
$y = -\frac{1}{\pi}(0) + 2 = 2$

Area $= \frac{1}{2}(1+2)(\pi)$
$= \frac{3}{2}\pi$

$= \int_0^\pi 2\cos\left(\frac{1}{2}x\right) + 1$

$= \left[2 \times 2\sin\left(\frac{1}{2}x\right) + x\right]_0^\pi$

$= \left(4\sin\left(\frac{1}{2}\pi\right) + \pi\right) - \left(4\sin(0) + 0\right)$

$= 4 + \pi$

$R = 4 + \pi - \frac{3}{2}\pi = \underline{\underline{4 - \frac{\pi}{2}}}$

4a) $x = t^2$

$\frac{dx}{dt} = \underline{\underline{2t}}$

4b) When $x = 0$, $0 = t^2$

$\underline{\underline{t = 0}}$

4c) When $x = 4$, $4 = t^2$

$t = 2$ or -2

$t \neq -2$ because $t \geq 0$.

11.8 - Finding areas

4c) $\int y\,dx = \int y \frac{dx}{dt}\,dt$

See my P2 Chp11.7.5 video on integrating parametric equation

x	0	4
t	0	2

$= \int (2t+1)(2t)\,dt$

$= \int 4t^2 + 2t\,dt = \frac{4t^3}{3} + t^2 + c$

$\text{Area} = \left[\frac{4t^3}{3} + t^2\right]_0^2$

$= \left(\frac{4(2)^3}{3} + 2^2\right) - (0) = \underline{\underline{\frac{44}{3}}}$

5a) $f(x) = \frac{1}{2}x\cos x$

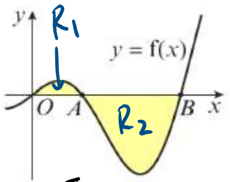
When $y=0$: $0 = \frac{1}{2}x\cos x$

$\frac{1}{2}x = 0$ or $\cos x = 0$

$x = 0$ or $x = \frac{1}{2}\pi, \frac{3\pi}{2}$

$\therefore A\left(\frac{\pi}{2}, 0\right),\ B\left(\frac{3\pi}{2}, 0\right)$

5b)



$R_1 = \int_0^{\frac{\pi}{2}} f(x)\,dx$

$= \int \frac{1}{2}x\cos x\,dx$

IBP (ILATE)

$u = \frac{1}{2}x \qquad v' = \cos x$

$u' = \frac{1}{2} \qquad v = \sin x$

$= \left[\frac{1}{2}x\sin x - \int \frac{1}{2}\sin x\,dx\right]_0^{\frac{\pi}{2}}$

$= \left[\frac{1}{2}x\sin x + \frac{1}{2}\cos x\right]_0^{\frac{\pi}{2}}$

$= \left[\frac{1}{2}\times\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right) + \frac{1}{2}\cos\left(\frac{\pi}{2}\right)\right] - \left[\frac{1}{2}\times 0\sin(0) + \frac{1}{2}\cos(0)\right]$

$= \frac{\pi}{4} + 0 - 0 - \frac{1}{2}$

$= \frac{\pi}{4} - \frac{1}{2}$

$R_2 = \left[\frac{1}{2}x\sin x + \frac{1}{2}\cos x\right]_{\frac{\pi}{2}}^{3\pi/2}$

$= \left(\frac{1}{2}\times\frac{3\pi}{2}\sin\left(\frac{3\pi}{2}\right) + \frac{1}{2}\cos\left(\frac{3\pi}{2}\right)\right) - \left(\frac{1}{2}\times\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right) + \frac{1}{2}\cos\left(\frac{\pi}{2}\right)\right)$

$= -\frac{3\pi}{4} + 0 - \left(\frac{\pi}{4}\right) = -\pi$

$\Rightarrow \text{Area} = \pi$

$\text{Total area} = \frac{\pi}{4} - \frac{1}{2} + \pi = \underline{\underline{\frac{5\pi}{4} - \frac{1}{2}}}$

11.8 - Finding areas

6a) When $y=0$, $0 = 4\sin t$

$\sin t = 0$

$t = \cancel{0}, \pi$

When $t = \pi$, $x = \pi^2$

$\therefore \underline{\underline{P(\pi^2, 0)}}$

6b) $\text{Area} = \int_0^{\pi^2} y \, dx$

x	0	π^2
t	0	π

$= \int_0^{\pi} y \frac{dx}{dt} \, dt$

$= \int_0^{\pi} (4\sin t)(2t) \, dt$

$= \int_0^{\pi} 8t\sin t \, dt$

IBP (ILATE)

$u = 8t \quad v' = \sin t$

$u' = 8 \quad v = -\cos t$

$= \left[8t(-\cos t) - \int (-\cos t)(8) \, dt \right]_0^{\pi}$

$= \left[-8t\cos t + 8\sin t \right]_0^{\pi}$

$= -8\pi\cos(\pi) + 8\sin(\pi) - (0 + 8\sin(0))$

$= \underline{\underline{8\pi}}$

11.8 - Finding areas

7a) $y = \sin x$; $y = -\cos 2x + 1$

$$\sin x = -\cos 2x + 1$$

$$\sin x = -(1 - 2\sin^2 x) + 1$$

$$0 = -1 + 2\sin^2 x + 1 - \sin x$$

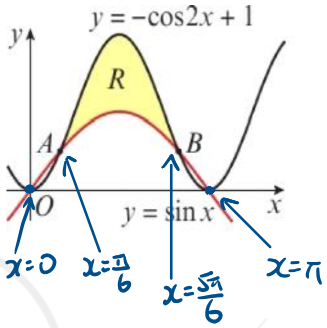
$$2\sin^2 x - \sin x = 0$$

$$\sin x(2\sin x - 1) = 0$$

$\sin x = 0$ or $2\sin x - 1 = 0$

$x = 0, \pi$ $\quad\quad$ $\sin x = \frac{1}{2}$

$\quad\quad\quad\quad x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$



$\therefore$ x-coord of B is $\frac{5\pi}{6}$.

7b) Area of $R = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \underbrace{-\cos 2x + 1}_{(y_1)} - \underbrace{\sin x}_{(y_2)} \, dx$

R is between the $y_1 = -\cos 2x + 1$ and $y = \sin x$.
$y_2 = -\cos 2x + 1$ is on top,
so area $= y_1 - y_2$

$$= \left[-\frac{1}{2}\sin 2x + x + \cos x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

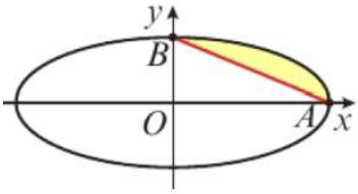
$$= \left[-\frac{1}{2}\sin\left(2\times\frac{5\pi}{6}\right) + \frac{5\pi}{6} + \cos\left(\frac{5\pi}{6}\right)\right] - \left[-\frac{1}{2}\sin\left(\frac{2\pi}{6}\right) + \frac{\pi}{6} + \cos\left(\frac{\pi}{6}\right)\right]$$

$$= \left(\frac{\sqrt{3}}{4} + \frac{5\pi}{6} - \frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{3}}{4} + \frac{\pi}{6} + \frac{\sqrt{3}}{2}\right)$$

$$= \frac{-\sqrt{3}}{2} + \frac{2}{3}\pi$$

11.8 - Finding areas

8) $x = 12\cos\theta$; $y = 5\sin\theta$



Trying to find Coords of A

When $y = 0$: $0 = 5\sin\theta$

$\sin\theta = 0$

$\theta = 0, \pi, 2\pi$

When $\theta = 0$, $x = 12\cos(0) = 12$

When $\theta = \pi$, $x = 12\cos(\pi) = -12$ (rej.)

$\longrightarrow$ Coords of A $= (12, 0)$ when $\theta = 0$

Trying to find Coords of B

When $x = 0$, $0 = 12\cos\theta$

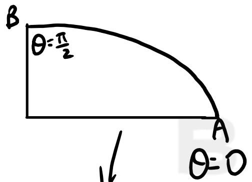
$\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

When $\theta = \frac{\pi}{2}$, $y = 5\sin\left(\frac{\pi}{2}\right) = 5$ $\longrightarrow$ $B(0, 5)$ when $\theta = \frac{\pi}{2}$

$\theta = \frac{3\pi}{2}$, $y = 5\sin\left(\frac{3\pi}{2}\right) = -5$ (rej.)

Shaded area =



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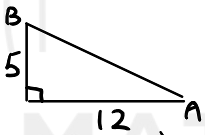

$= \frac{12 \times 5}{2} = 30$

$$= \int_0^{\frac{\pi}{2}} y \frac{dx}{d\theta} \, d\theta$$

$$= \int 5\sin\theta(-12\sin\theta) \, d\theta$$

$\cos 2A \equiv 1 - 2\sin^2 A$

$2\sin^2 A \equiv 1 - \cos 2A$

$\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$

$$= -60 \int \sin^2\theta \, d\theta$$

$$= -60 \int \frac{1}{2} - \frac{1}{2}\cos 2\theta$$

$$= -60 \left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \right]_0^{\frac{\pi}{2}}$$

$$= -60 \left[\left(\frac{1}{2} \times \frac{\pi}{2} - \frac{1}{4}\sin\left(\frac{2\pi}{2}\right) \right) - \left(\frac{1}{2} \times 0 - \frac{1}{4}\sin(0) \right) \right]$$

$= -15\pi \Rightarrow$ Area $= 15\pi$

-ve because the graph is sketched from right to left as θ increases.

Shaded area $= 15\pi - 30$