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Ch: 12 → Vectors

SET A

BRONZE:

$A\begin{pmatrix}8\\-6\\4\end{pmatrix}$ $\quad B\begin{pmatrix}k\\-4\\-7\end{pmatrix}$ $\quad \overrightarrow{AB} = 15$ $\quad d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$15 = \sqrt{(8-k)^2+(-6+4)^2+(4+7)^2} \Rightarrow 225 = (8-k)^2+4+121 \Rightarrow (8-k)^2 = 100$

$\Rightarrow 8-k = \pm 10 \Rightarrow k = 8+10 = 18$ or $k = 8-10 = -2$

SILVER:

$a = 4pi - 5pj + 3pk$, $|a| = 10\sqrt{6}$ $\quad |a| = \sqrt{x^2+y^2+z^2}$

$\Rightarrow 10\sqrt{6} = \sqrt{(4p)^2+(-5p)^2+(3p)^2} \Rightarrow 600 = 16p^2+25p^2+9p^2 \Rightarrow \cancel{600}^{12} = \cancel{50}p^2 \quad p = \pm\sqrt{12} = \pm 2\sqrt{3}$

GOLD:

$m = 3 \quad F_1 = \begin{pmatrix}4\\q\\2\end{pmatrix} \quad F_2 = \begin{pmatrix}-q\\2\\-1\end{pmatrix} \quad F_3 = \begin{pmatrix}-1\\3\\-q\end{pmatrix} \quad a = 2$

$R_F = F_1+F_2+F_3 \Rightarrow \begin{pmatrix}4\\q\\2\end{pmatrix} + \begin{pmatrix}-q\\2\\-1\end{pmatrix}\begin{pmatrix}-1\\3\\-q\end{pmatrix} = \begin{pmatrix}3-q\\5+q\\1-q\end{pmatrix}$ $\quad |R| = \sqrt{(3-q)^2+(5+q)^2+(1-q)^2}$

$F = ma \Rightarrow \sqrt{(3-q)^2+(5+q)^2+(1-q)^2} = 3(2)$

$\Rightarrow 36 = (3-q)^2+(5+q)^2+(1-q)^2$

$\Rightarrow 36 = 9 - 6q + q^2 + 25 + 10q + q^2 + 1 + q^2 - 2q$

$\Rightarrow 1 = 3q^2 + 2q \Rightarrow 3q^2+2q-1 = 0 \Rightarrow (q+1)(3q-1) = 0$

$q = \frac{1}{3}$ or $q = -1$

SET B

BRONZE

$A\begin{pmatrix}7\\4\\-1\end{pmatrix}$ $B\begin{pmatrix}11\\-2\\3\end{pmatrix}$ $C\begin{pmatrix}6\\-15\\16\end{pmatrix}$ $D\begin{pmatrix}-2\\-3\\8\end{pmatrix}$

a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}11\\-2\\3\end{pmatrix} - \begin{pmatrix}7\\4\\-1\end{pmatrix} = \begin{pmatrix}4\\-6\\4\end{pmatrix} = 4i - 6j + 4k$

$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD} = \begin{pmatrix}6\\-15\\16\end{pmatrix} - \begin{pmatrix}-2\\-3\\8\end{pmatrix} = \begin{pmatrix}4\\-12\\8\end{pmatrix} = 8i - 12j + 8k$

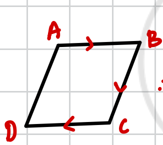
b) $\overrightarrow{AB} \cdot 2(4i - 6j + 4k) = 8i - 12j + 8k = \overrightarrow{DC}$ $\left[\overrightarrow{DC} = 2\overrightarrow{AB}\right]$

c) $|AB| = k|DC| \Rightarrow k = \frac{1}{2}$

d) quadrilateral ABCD is a trapezium.

SILVER:

$A\begin{pmatrix}6\\-8\\4\end{pmatrix}$ $B\begin{pmatrix}-3\\-4\\2\end{pmatrix}$ $C\begin{pmatrix}-7\\9\\-3\end{pmatrix}$



$\therefore AB = CD$
$AD = BC$

Let D be $\begin{pmatrix}x\\y\\z\end{pmatrix}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}-3\\-4\\2\end{pmatrix} - \begin{pmatrix}6\\-8\\4\end{pmatrix} = \begin{pmatrix}-9\\4\\-2\end{pmatrix}$

$\begin{pmatrix}-9\\4\\-2\end{pmatrix} = \begin{pmatrix}-7+x\\9-y\\-3-z\end{pmatrix}$

$\Rightarrow -9 = -7 + x \Rightarrow \boxed{x = 2}$

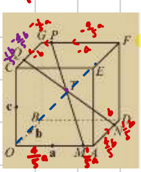
$\Rightarrow 4 = 9 - y \Rightarrow \boxed{y = 5}$

$\Rightarrow -2 = -3 - z \Rightarrow \boxed{z = -1}$

$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD} = \begin{pmatrix}-7\\9\\-3\end{pmatrix} - \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}-7+x\\9-y\\-3-z\end{pmatrix}$

$D\begin{pmatrix}2\\5\\-1\end{pmatrix}$

GOLD:



$OT_1 = \overrightarrow{OM} + \overrightarrow{MT}\,(\lambda\,\overrightarrow{MP})$

$\Rightarrow \overrightarrow{MP} \cdot \frac{1}{5}a + b + c - \frac{4}{5}a \Rightarrow -\frac{3}{5}a + b + c$

$OT_1 = \frac{4}{5}a + \lambda\left(-\frac{3}{5}a + b + c\right)$

$OT_1 = \frac{4}{5}a - \lambda\frac{3}{5}a + \lambda b + \lambda c$

$OT_1 = \frac{1}{5}a(4 - 3\lambda) + \lambda b + \lambda c$

$OT_2 = \overrightarrow{OA} + \overrightarrow{AN} + \overrightarrow{NT}\,(\mu\,\overrightarrow{NQ})$

$\Rightarrow \overrightarrow{NQ} = \frac{1}{5}b + c - a - \frac{4}{5}b = -\frac{3}{5}b - a + c$

$OT_2 = a + \frac{4}{5}b + \mu\left(-\frac{3}{5}b - a + c\right)$

$OT_2 = a + \frac{4}{5}b + -\frac{3}{5}\mu b - \mu a + \mu c$

$OT_2 = a(1 - \mu) + \frac{1}{5}b(4 - 3\mu) + \mu c$

Equate : $OT_1 = OT_2$ $\boxed{\lambda = \mu}$

$\frac{4}{5} - \frac{3}{5}\lambda = 1 - \mu \Rightarrow \frac{4}{5} - 1 = \frac{3}{5}\mu - 1\mu$

$\Rightarrow -\frac{1}{8} = -\frac{2}{5}\mu \Rightarrow \mu = \frac{1}{2}$

$\lambda = \frac{1}{2}$; $\lambda = \mu$

$\therefore$ That MP and NQ intersect at T which is the midpoint of each line and therefore MP and NQ bisect each other at T.