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9.8 Implicit Differentiation

1] $u = y^5$

$\frac{du}{dy} = 5y^4$

$\frac{d}{dx}(y^5) \Rightarrow \frac{du}{dx} = \frac{du}{dy} \times \frac{dy}{dx}$

$\frac{du}{dx} = 5y^4 \times \frac{dy}{dx}$

$\therefore \frac{d}{dx}(y^5) = 5y^4 \frac{dy}{dx}$

2] $\frac{d}{dx}(x^3y^2)$

$u = x^3$ $\quad v = y^2$

$\frac{du}{dx} = 3x^2$ $\quad \frac{dv}{dy} = 2y \frac{dy}{dx}$

$\frac{du}{dv} = 3x^2y^2 + 2yx^3 \frac{dy}{dx}$

So $\frac{d}{dx}(x^3y^2) = 2x^3y \frac{dy}{dx} + 3x^2y^2$

3a] $x^4 - y^3 = 3$

$4x^3 - 3y^2 \frac{dy}{dx} = 0$

$+3y^2 \frac{dy}{dx} = +4x^3$

$\frac{dy}{dx} = \frac{4x^3}{3y^2}$

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3b] $3x^2 + 4y^2 - 5x^2y = 2$

$u = -5x^2$ $\quad v = y$

$\frac{du}{dx} = -10x$ $\quad \frac{dv}{dy} = \frac{dy}{dx}$

$\frac{du}{dv} = -10xy - 5x^2\frac{dy}{dx}$

$= 6x + 8y\frac{dy}{dx} - 10xy - 5x^2\frac{dy}{dx} = 0$

$= 8y\frac{dy}{dx} - 5x^2\frac{dy}{dx} = -6x + 10xy$

$\frac{dy}{dx}(8y - 5x^2) = -6x + 10xy$

$\frac{dy}{dx} = \frac{-6x + 10xy}{8y - 5x^2}$

$\frac{dy}{dx} = \frac{6x - 10xy}{5x^2 - 8y}$

c] $y = \frac{5y^2}{x^2 - y^2}$

$u = x^2$ $\quad v = y$

$\frac{du}{dx} = 2x$ $\quad \frac{dv}{dy} = \frac{dy}{dx}$

$y(x^2 - y^2) = 5y^2$

$x^2y - y^3 = 5y^3$

$\frac{du}{dv} = x^2\frac{dy}{dx} + 2xy - 3y^2\frac{dy}{dx} = 10y\frac{dy}{dx}$

$= x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} - 10y\frac{dy}{dx} = -2xy$

$\frac{dy}{dx}(x^2 - 3y^2 - 10y) = -2xy$

$\frac{dy}{dx} = \frac{-2xy}{x^2 - 3y^2 - 10y}$

$\frac{dy}{dx} = \frac{2xy}{-x^2 + 3y^2 + 10y}$

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3d] $4\ln(x-5) - \ln(y^3) = 0$

$u = 4\ln(x-5)$

$\frac{du}{dx} = 4 \times \frac{1}{x-5} \times 1$

$\frac{du}{dx} = \frac{4}{x-5}$

$v = \frac{1}{y^3} \times 3y^2 \frac{dy}{dx}$

$\frac{dv}{dy} = \frac{3}{y} \frac{dy}{dx}$

$\Rightarrow \frac{4}{x-5} - \frac{3}{y}\frac{dy}{dx} = 0$

$= \frac{3}{y}\frac{dy}{dx} = \frac{4}{x-5}$

$= \frac{dy}{dx} = \frac{\frac{4}{x-5}}{\frac{3}{y}}$

$= \frac{dy}{dx} = \frac{4}{x-5} \times \frac{y}{3}$

$= \frac{dy}{dx} = \frac{4y}{3(x-5)}$

e] $\cos x + \sin^2 y = 1$

$= (-\sin x) + 2\sin y \cos y \frac{dy}{dx} = 0$

$= 2\sin y \cos y \frac{dy}{dx} = \sin x$

$= \frac{dy}{dx}(2\sin y \cos y) = \sin x$

$= \frac{dy}{dx} = \frac{\sin x}{2\sin y \cos y}$

$(\sin y)^2$

$= 2(\sin y)(\cos y \frac{dy}{dx}$

$= 2\sin y \frac{dy}{dx} \cos$

$= 2\sin y \cos y \frac{dy}{dx}$

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3b] $xe^{-y} + ye^{2x} - 7 = 0$

$u = x \qquad v = e^{-y}$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dy} = -e^{-y}\frac{dy}{dx}$

$\frac{du}{dv} = -xe^{-y}\frac{dy}{dx} + e^{-y}$

$u = y \qquad v = e^{2x}$

$\frac{du}{dy} = \frac{dy}{dx} \qquad \frac{dv}{dx} = 2e^{2x}$

$\frac{du}{dv} = e^{2x}\frac{dy}{dx} + 2ye^{2x}$

$-xe^{-y}\frac{dy}{dx} + e^{-y} + e^{2x}\frac{dy}{dx} + 2ye^{2x} = 0$

$-xe^{-y}\frac{dy}{dx} + e^{2x}\frac{dy}{dx} = -2ye^{2x} - e^{-y}$

$\frac{dy}{dx}\left(e^{2x} - xe^{-y}\right) = -2ye^{2x} - e^{-y}$

$\frac{dy}{dx} = \frac{-2ye^{2x} - e^{+y}}{e^{2x} - xe^{-y}}$

$\frac{dy}{dx} = \frac{2ye^{2x} + e^{-y}}{xe^{-y} - e^{2x}}$

4a] $x^2 - 4x^2y + y^2 = 21 \qquad u = -4x^2 \qquad v = y$

$\frac{du}{dx} = -8x \qquad \frac{dv}{dy} = \frac{dy}{dx}$

$\frac{du}{dv} = -8xy - 4x^2\frac{dy}{dx}$

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4a] Cont.

$$2x - 8xy - 4x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$-4x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x + 8xy$$

$$\frac{dy}{dx}(-4x^2 + 2y) = -2x + 8xy$$

$$\frac{dy}{dx} = \frac{8xy - 2x}{2y - 4x^2}$$

b] Sub points $(2, -1)$ in $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{8(2)(-1) - 2(2)}{2(-1) - 4(2)^2}$$

$$\frac{dy}{dx} = \frac{10}{9}$$

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c] $y - y_1 = m(x - x_1)$

$$y - (-1) = \frac{10}{9}(x - 2)$$

$$y = \frac{10x}{9} - \frac{20}{9} - 1$$

$$y = \frac{10x}{9} - \frac{29}{9}$$

5] $3x^2 - y^3 - 5xy = 1$ $\quad u = -5x \quad v = y$

$$\frac{du}{dx} = -5 \qquad \frac{dv}{dy} = \frac{dy}{dx}$$

$$\frac{du}{dv} = -5y - 5x\frac{dy}{dx}$$

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5] Cont.

$$= 6x - 3y^2\frac{dy}{dx} - 5y - 5x\frac{dy}{dx} = 0$$

$$(-3y^2 - 5x)\frac{dy}{dx} = 5y - 6x$$

$$\frac{dy}{dx} = \frac{5y - 6x}{-3y^2 - 5x}$$

Sub points $(2, 1)$ in $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{5(1) - 6(2)}{-3(1)^2 - 5(2)}$$

$$\frac{dy}{dx} = \frac{7}{13}$$

6] $e^{-3y} - e^{-2x} = 4xy$

$u = 4x$ $\quad v = y$

$\frac{du}{dx} = 4$ $\quad \frac{dv}{dy} = \frac{dy}{dx}$

$$\frac{du}{dv} = 4y + 4x\frac{dy}{dx}$$

$$= -3e^{-3y}\frac{dy}{dx} + 2e^{-2x} = 4y + 4x\frac{dy}{dx}$$

$$= -3e^{-3y}\frac{dy}{dx} - 4x\frac{dy}{dx} = 4y - 2e^{-2x}$$

$$\frac{dy}{dx}(-3e^{-3y} - 4x) = 4y - 2e^{-2x}$$

$$\frac{dy}{dx} = \frac{4y - 2e^{-2x}}{-3e^{-3y} - 4x}$$

$$\frac{dy}{dx} = \frac{2e^{-2x} - 4y}{4x + 3e^{-3y}}$$

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7a] $\cos 2x + \sin 2y = 1$ ——— (1)

$2(-\sin 2x) + 2(\cos 2y)\frac{dy}{dx} = 0$

$2\cos 2y \frac{dy}{dx} = 2\sin 2x$

$\frac{dy}{dx} = \frac{2\sin 2x}{2\cos 2y}$

$\frac{dy}{dx} = \frac{\sin 2x}{\cos 2y}$

b] Sub $x = -\frac{\pi}{6}$ in ——— (1)

$\cos 2\left(-\frac{\pi}{6}\right) + \sin 2y = 1$

$\frac{1}{2} + \sin 2y = 1$

$\sin 2y = \frac{1}{2}$

$2y = \frac{\pi}{6}$

$y = \frac{\pi}{12}$

Sub $x = -\frac{\pi}{6}$ & $y = \frac{\pi}{12}$ in $\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{\sin 2\left(-\frac{\pi}{6}\right)}{\cos 2\left(\frac{\pi}{12}\right)}$

$\frac{dy}{dx} = -1$

$y - y_1 = m(x - x_1)$

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7b] Cont.

$$y - \frac{\pi}{12} = -1\left(x + \frac{\pi}{6}\right)$$

$$y = -x - \frac{\pi}{6} + \frac{\pi}{12}$$

$$y = -x - \frac{\pi}{12}$$

$$x + y + \frac{\pi}{12} = 0$$

8] $4x^2 + y^2 + 6x - 8y - 10 = 0$

$$8x + 2y\frac{dy}{dx} + 6 - 8\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} - 8\frac{dy}{dx} = -8x - 6$$

$$\frac{dy}{dx}(2y - 8) = -8x - 6$$

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$$\frac{dy}{dx} = \frac{-8x - 6}{2y - 8}$$

Sub $(1, 8)$ in $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-8(1) - 6}{2(8) - 8}$$

$$\frac{dy}{dx} = -\frac{7}{4}$$

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

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8] Cont.

$$y - 8 = \frac{4}{7}(x - 1)$$

$$y = \frac{4x}{7} - \frac{4}{7} + 8$$

$$y = \frac{4x}{7} + \frac{52}{7}$$

$$7y = 4x + 52$$

$$4x - 7y + 52 = 0$$

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