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7.7 : Modelling with trigonometric functions

① a) $h = 0.3 - 0.3\cos(240t)^\circ$

$\cos(240t)^\circ = -1 \quad = 0.3 - -0.3 = 0.6\,m$

b) $\cos(240t)^\circ = -1 \qquad 240t = 180 \qquad t = \frac{3}{4}$ sec

c) $T = \frac{360^\circ}{240^\circ} = 1.5$ sec No. of somersaults $= \frac{30}{1.5} = 20$

d) A body moving in the air is better modelled by a parabola.

② $d = 40.5 + 2.5\sin(\pi t)$

a) $d = 40.5 + 2.5\sin(3\pi)$; $t = 3$ $d = 40.5$ cm

b) $\sin \pi t = 1$; $40.5 + 2.5(1) = 43$ cm

c) $\sin \pi t = 1 \qquad \pi t = \frac{\pi}{2}, \frac{5}{2}\pi, \frac{9}{2}\pi \qquad t = 0.5, 2.5, 4.5$

d) min depth is $(d = 40.5 - 2.5)$ $d = 38$ cm.

$38 < 40$, so the water stream is not deep enough

③ $P = 105 + 25\sin(450t)^\circ$

a) $P = 105 + 25 \qquad P = 130$ mm Hg $(\sin(450t)^\circ = 1)$

b) $P = 105 - 25 \qquad P = 80$ mm Hg $(\sin(450t)^\circ = -1)$

c) $t = 4$; $P = 105 + 25\sin(450(4)) = 105$ mm Hg

d) $P = 110$ mmHg $\quad 110 = 105 + 25\sin(450t)$

$\Rightarrow \quad 5 = 25\sin(450t) \quad \Rightarrow \quad \frac{1}{5} = \sin(450t)$

$\Rightarrow \quad 450t = \arcsin\left[\frac{1}{5}\right] \quad 450t = (11.54, 168.46, 371.54, 528.46, 731.54, 888.46)$

divide by 450 to get t

$t = 0.03, 0.37, 0.83, 1.17, 1.63, 1.97$ seconds

④ $S = 6 - 3\sin(0.5t + 3), \quad 0 \le t \le 12$

a) $\sin(0.5t+3) = -1 \Rightarrow S = 6 - 3(-1) = 9$

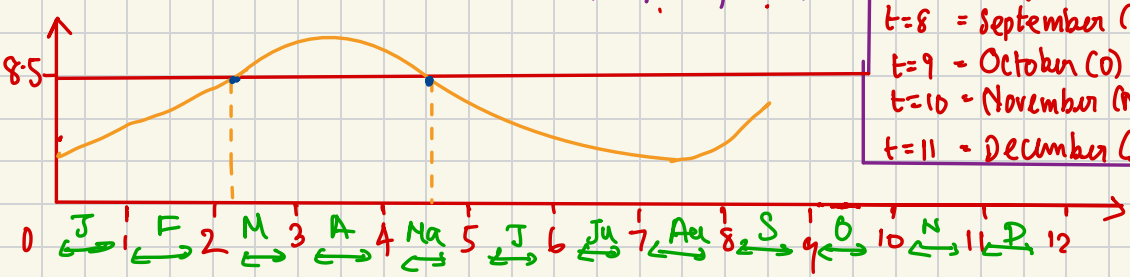
Max = 9,000 Month : April

b) Let $S = 8.5 \Rightarrow 8.5 = 6 - 3\sin(0.5t+3)$

$2.5 = -3\sin(0.5t+3) \Rightarrow 0.5t + 3 = \sin^{-1}\left(\frac{2.5}{3}\right)$

$\Rightarrow t = 2.26$ or 4.569 (March, April, May)

$t=0$ = January (J)
$t=1$ = February (F)
$t=2$ = March (M)
$t=3$ = April (A)
$t=4$ = May (Ma) $S = 6 - 3\sin(0.5t+3)$
$t=5$ = June (Ju)
$t=6$ = July (Ju)
$t=7$ = August (Au)
$t=8$ = September (S)
$t=9$ = October (O)
$t=10$ = November (N)
$t=11$ = December (D)



c) $t = 11$ (1st december 2018) $\quad t = 12$ (1st november 2019) Let $t = 12$

$S = 6 - 3\sin(0.5 \times 12 + 3) = 4.78$ thousand $= 4760$

d) Cannot tell because data only shows info in 2018. Any prediction outside data range is extrapolation

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⑤ $\frac{5\sqrt{2}}{2}\cos\theta - \frac{5\sqrt{2}}{2}\sin\theta = R\cos(\theta + a)$

$R = \sqrt{\left(\frac{5\sqrt{2}}{2}\right)^2 + \left(\frac{5\sqrt{2}}{2}\right)^2} = 5$

$\tan a = \frac{\frac{5\sqrt{2}}{2}}{\frac{5\sqrt{2}}{2}} = 1 \quad a = 45$

$= 5\cos(\theta + 45)$

b) i) $5\underbrace{\cos(\theta+45)^\circ}_{-1}$ min $= -5$

ii) $\cos(\theta+45)^\circ = -1$ $\quad \theta+45^\circ = 180^\circ$ $\quad \theta = 135^\circ$

c) $h = 9 - \frac{5\sqrt{2}}{2}\cos(30t) + \frac{5\sqrt{2}}{2}\sin(30t) = 9 - 5\cos(30t+45^\circ)$

Max:

$= \cos(30t+45^\circ) = -1 = 9-5(-1) = 9+5 = 14\text{m}$

$= 30t+45^\circ = 180^\circ \quad t = \frac{135^\circ}{30^\circ} = 4{\cdot}5 = 12{:}00 + 4{:}30 = 4{:}30$

$= 14\text{m}$ at $4{:}30$

d) Min:

$\cos(30t+45^\circ) = 1 = 9-5(1) = 9-5 = 4\text{m}$

$= 30t+45^\circ = 360^\circ \quad t = 10{\cdot}5 = 12{:}00 + 10{:}30 = 22{:}30$

$= 4\text{m}$ at $22{:}30$

e) i) $h > 9 = 9 - 5\cos(30t+45^\circ) > 9 \quad \cos(30t+45^\circ) < 0$

$30t+45^\circ = 90^\circ \quad t = \frac{45}{30} = 1{\cdot}5 = 1{:}30$

ii) $h < 9$; $9 - 5\cos(30t+45^\circ) < 9 \quad \cos(30t+45^\circ) > 0$

$30t+45^\circ = 270^\circ \quad t = \frac{225}{30} = 7{\cdot}5 = 7{:}30$

⑥ $0{\cdot}2\cos x + 0{\cdot}35\sin x = R\cos(x-a)$

a) $R = \sqrt{0{\cdot}2^2 + 0{\cdot}35^2} = \frac{\sqrt{65}}{20} \quad \tan a = \frac{0{\cdot}35}{0{\cdot}2} \quad a = 1{\cdot}1$

$= \frac{\sqrt{65}}{20}\cos(x-1{\cdot}1)$

b) $T = 30 + \frac{\sqrt{65}}{20}\cos(2m-1{\cdot}1)$; $m=1 = 30{\cdot}25^\circ\text{C}$

c) Max = $\cos(2m - 1.1) = 1 = 30 + \frac{\sqrt{65}}{20} = \frac{600 + \sqrt{65}}{20}$

Min = $\cos(2m - 1.1) = -1 = 30 - \frac{\sqrt{65}}{20} = \frac{600 - \sqrt{65}}{20}$

Max - Min = $\frac{600 + \sqrt{65}}{20} - \frac{600 + \sqrt{65}}{20} = \frac{2\sqrt{65}}{20} = \frac{\sqrt{65}}{10} = 0.8°C$

d) $\cos(2m - 1.1) = 1 \Rightarrow 2m - 1.1 = 0$

$\Rightarrow$ $2m - 1.1 = 0, 2\pi, 4\pi$ $m = 0.6, 3.7, 6.8$ minutes (2dp)

⑦ a) $132\sin x - 72\cos x = R\sin(x - a)$

$\sqrt{132^2 + 72^2} = 12\sqrt{157}$ $\tan a = \frac{72}{132}$ $a = 0.5$

$= 12\sqrt{157}(x - 0.5)$

$P = 200 + 12\sqrt{157}\sin(0.8t - 0.5)$

b) $\sin(0.8t - 0.5) = 1 \Rightarrow 200 + 12\sqrt{157} = 350$ (3sf)

$0.8t - 0.5 = \frac{\pi}{2}$ $t = 3$ 350p in March

c) $\sin(0.8t - 0.5) = -1 \Rightarrow 200 - 12\sqrt{157} = 50$ (2sf)

$0.8t - 0.5 = \frac{3}{2}\pi$ $\Rightarrow$ 50p in July

$t = 7$

d) $P = 62.24$ $t = 6$; $t = 5$ $P = 147.25$

So, it is June

e) $P(0) = 128$ $P(12) = 250$

Cost = $2000 \times 128 = 25600$ Revenue = $2000 \times 250 = 500000$

Profit = $500000 - 260000 = \frac{240000}{100} = £2400$