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Ch10: Numerical roots
SET-A

BRONZE:

$f(x) = \frac{1}{2}\cos(x^2) - \frac{1}{2}x + 1$

a) $f(1) = \frac{1}{2}\cos(1^2) - \frac{1}{2}(1) + 1 = 0.77.. > 0$ $\quad F(1.5) = \frac{1}{2}\cos(1.5^2) - \frac{1}{2}(1.5) + 1 = -0.064.. < 0$

$\therefore$ Sign change implies at least one root in interval.

b) $\frac{1}{2}\cos(x^2) = \frac{1}{2}x - 1 \Rightarrow \cos(x^2) = x - 2 \Rightarrow x^2 = \arccos(x-2) \Rightarrow x = \sqrt{\arccos(x-2)}$

c) $x_0 = 1.5$; $\quad x_{n+1} = \sqrt{\arccos(x_n - 2)}$

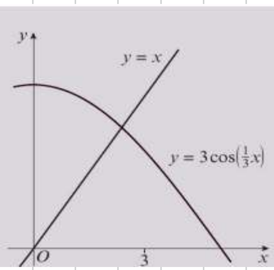
$x_1 = \sqrt{\arccos(x_0 - 2)} = 1.447$

$x_2 = \sqrt{\arccos(x_1 - 2)} = 1.469$

$x_3 = \sqrt{\arccos(x_2 - 3)} = 1.460$

SILVER:

$y = 3\cos\left(\frac{1}{3}x\right) \Rightarrow x = 3\cos\left(\frac{1}{3}x\right)$



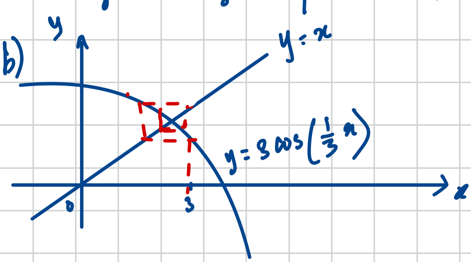
a) $x - 3\cos\left(\frac{1}{3}x\right) = 0$

$f(2) = 2 - 3\cos\left(\frac{1}{3}(2)\right) = -0.3576... < 0$

$f(3) = 3 - 3\cos\left(\frac{1}{3}(3)\right) = 1.3790... > 0$

$\therefore$ Sign change implies at least one root in interval.

b)



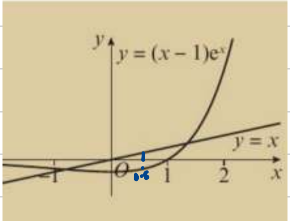
c) $x_{n+1} = 3\cos\left(\frac{1}{3}x_n\right) \Rightarrow x_1 = 3\cos\left(\frac{1}{3}x_0\right) = 1.621$

$x_2 = 3\cos\left(\frac{1}{3}x_1\right) = 2.573$

$x_3 = 3\cos\left(\frac{1}{3}x_2\right) = 1.963$

GOLD:

$y = (x-1)e^x \Rightarrow x = (x-1)e^x \Rightarrow x-(x-1)e^x$



a) $f(1) = 1-(1-1)e^1 = 1 > 0$

$f(2) = 2-(2-1)e^2 = -5.389\ldots < 0$

$\therefore$ Sign change implies at least one root in interval

b) i) $x_0 = \frac{1}{2} \Rightarrow$ The iteration will converge to the other root, a.

ii) $x_0 = 2$, the iteration will diverge.

c) $a = -0.806 \Rightarrow -0.8055 \leq x \leq -0.8065$

$f(-0.8055) = 0.0013 > 0$, $f(-0.8065) = -0.00004\ldots < 0$

$\therefore$ Sign change implies at least one root in interval, so $a = -0.806$ (3dp)

SET B

BRONZE:

$f(x) = -\frac{4}{3}x + 3x^{\frac{4}{3}} - 2$

a) $f(1.1) = -\frac{4}{3}(1.1) + 3(1.1)^{\frac{4}{3}} - 2 = -0.0601\ldots < 0$

$f(1.2) = -\frac{4}{3}(1.2) + 3(1.2)^{\frac{4}{3}} - 2 = 0.2255\ldots > 0$

$\therefore$ Sign change implies at least one root in interval

b) $x_0 = 1.2 \Rightarrow x_1 = 1.2 - \frac{f(1.2)}{f'(1.2)} = 1.12$ (2dp)

c) $f(1.115) = -\frac{4}{3}(1.115) + 3(1.115)^{\frac{4}{3}} - 2 = -0.018\ldots < 0$

$f(1.125) = -\frac{4}{3}(1.125) + 3(1.125)^{\frac{4}{3}} - 2 = 0.010\ldots > 0$

$\therefore$ Sign change implies at least one root in interval, so part b answer is correct to 2dp (1.12)

SILVER:

$f(x) = e^{-x}\sin(x^2)$

a) $f(1.5) = e^{-1.5}\sin(1.5^2) = 0.1736\ldots > 0$

$f(2) = e^{-2}\sin(2^2) = -0.1024\ldots < 0$

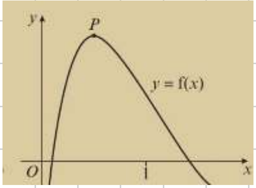
$\therefore$ Sign change implies at least one root in interval.

b) $x_0 = 1.75 \Rightarrow x_1 = 1.75 - \frac{f(1.75)}{f'(1.75)} = 1.772$ (3dp)

c) $f(1.7715) = 0.0005\ldots > 0$, $f(1.7725) = -0.00002\ldots < 0$

$\therefore$ Sign change implies at least one root in interval, so part b answer is correct to 3dp.

GOLD:



$f(x) = 12xe^{-2x} - 1$

a) $f'(x) = 12e^{-2x} + 12x(-2e^{-2x}) = 12e^{-2x}(1-2x)$

$f'(x) = 0 \Rightarrow 12e^{-2x}(1-2x) = 0$

$x = \frac{1}{2}$, $p = \frac{1}{2}$

$q = f\left(\frac{1}{2}\right) = 12\left(\frac{1}{2}\right)e^{-2\left(\frac{1}{2}\right)} - 1 = 6e^{-1} - 1 = \frac{6}{e} - 1$

$\Rightarrow P\left(\frac{1}{2}, \frac{6}{e} - 1\right)$

b) $f'(p) = 0$, so would mean dividing by zero in the Newton-Raphson formula, which is not valid.

c) $x_0 = 1.5$; $x_1 = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.41$ (2dp)