Author: <u>Naga Karthik</u>

This step-by-step solution guide has been created by **<u>Naga Karthik</u>** for educational purposes. While we have made every effort to ensure the accuracy of the information presented, it is possible that there may be errors or omissions. We encourage users to critically evaluate and verify the content. BF Maths and the author cannot be held responsible for any errors or inaccuracies in this guide.

If you find any mistakes or have any suggestions for improvements, please contact us at bfmathshello@gmail.com. Your feedback is invaluable in helping us maintain the quality and accuracy of our resources. Please specify *which exercise and which question* in the email.

Thank you for using BF Maths for your maths revision!

CH-7: TRIGONOMETRY AND MODELLING

7.1 → Addition Formulae

① a) $\sin[(90^\circ - A) + B] = \sin(90^\circ - A)\cos B + \cos(90^\circ - A)\sin B$

$\Rightarrow \sin(90^\circ - A) = \cos A$

$\Rightarrow \cos(90^\circ - A) = \sin A$

$\Rightarrow \cos A \cos B + \sin A \sin B = \cos(A - B)$

b) $x = A \quad y = -B \quad \Rightarrow \cos(A+B) = \cos(A+B)$

$\Rightarrow \cos(A - (-B)) = \cos A \cos(-B) + \sin A \sin(-B)$

$\Rightarrow \cos(-B) = \cos B$

$\Rightarrow \sin(-B) = -\sin B$

$\cos A \cos B - \sin A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

② a) $\sin\left[x - \frac{\pi}{3}\right] \Rightarrow \sin(A-B) = \sin A \cos B - \sin B \cos A.$

$\Rightarrow \sin(x)\cos\left[\frac{\pi}{3}\right] - \sin\left[\frac{\pi}{3}\right]\cos(x)$

$\Rightarrow \sin x\left[\frac{1}{2}\right] - \frac{\sqrt{3}}{2}\cos x \Rightarrow \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x$

$a = \frac{1}{2} \quad b = -\frac{\sqrt{3}}{2} \Rightarrow a\sin x + b\cos x.$

b) $\cos\left[x - \frac{\pi}{6}\right] \Rightarrow \cos(A-B) = \cos A \cos B + \sin A \sin B$

$\Rightarrow \cos(x)\cos\left[\frac{\pi}{6}\right] + \sin(x)\sin\left[\frac{\pi}{6}\right]$

$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = p\cos x + q\sin x$

$p = \sqrt{3}/2 \quad q = \frac{1}{2}$

c) $\tan\left(x+\frac{\pi}{3}\right) \Rightarrow \dfrac{\tan x + m}{1 - m\tan x}$

$$\tan[A+B] = \frac{\tan A + \tan B}{1 - \tan A \tan B} \Rightarrow \frac{\tan x + \tan\left(\frac{\pi}{3}\right)}{1 - \tan\left[\frac{\pi}{3}\right]\tan x}$$

$$\Rightarrow \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x} \qquad \boxed{m = \sqrt{3}}$$

③ a) $\dfrac{\tan 7\theta - \tan 3\theta}{1 + \tan 7\theta \tan 3\theta} = \tan(A-B) \Rightarrow \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\Rightarrow A = 7\theta \quad B = 3\theta \Rightarrow \tan[7\theta - 3\theta] = \tan 4\theta$$

b) $\cos\left[\dfrac{3a+5b}{3}\right]\cos\left[\dfrac{3a-5b}{3}\right] - \sin\left[\dfrac{3a+5b}{3}\right]\sin\left[\dfrac{3a-5b}{3}\right]$

$\cos(A+B) = \cos A\cos B - \sin A \sin B$

$A = \dfrac{3a+5b}{3} \qquad B = \dfrac{3a-5b}{3}$

$$\Rightarrow \cos\left[\frac{3a+5b}{3} + \frac{3a-5b}{3}\right] = \cos\left[\frac{\cancel{6}^{2}a}{\cancel{3}}\right] = \cos 2a$$

c) $\dfrac{\sqrt{2}}{2}\sin x - \dfrac{\sqrt{2}}{2}\cos x \Rightarrow \sin(A-B) = \sin A\cos B - \sin B\cos A$

$A = x \quad B = 45^\circ \qquad \sin 45^\circ = \dfrac{\sqrt{2}}{2}$

$\Rightarrow \sin(x - 45^\circ) \qquad \cos 45^\circ = \dfrac{\sqrt{2}}{2}$

④ $\dfrac{\cos(x-y)}{\sin(x+y)} \Rightarrow \left(\dfrac{\cos x\cos y + \sin x\sin y}{\sin x\cos y + \sin y\cos x}\right)$ divide by $\cos x\cos y$

$$\Rightarrow \frac{\dfrac{\cancel{\cos x\cos y}}{\cancel{\cos x\cos y}} + \dfrac{\sin x\sin y}{\cos x\cos y}}{\dfrac{\sin x\cancel{\cos y}}{\cos x\cancel{\cos y}} + \dfrac{\sin y\cancel{\cos x}}{\cancel{\cos x}\cos y}} \Rightarrow \frac{1 + \tan x\tan y}{\tan x + \tan y}$$

$\boxed{\tan x = \dfrac{\sin x}{\cos x}}$

⑤ a) $\frac{1}{2}(\cos x - \sqrt{3}\sin x) = \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

$\cos(A-B) = \cos A\cos B - \sin B \sin A$

$\Rightarrow \cos\left(x + \frac{\pi}{3}\right) \Rightarrow \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

$\sin 30^\circ = \frac{\sqrt{3}}{2}$

$\cos 30^\circ = \frac{1}{2} \Rightarrow \cos\left[x + \frac{\pi}{3}\right]$

b) $\frac{1}{2}(\sqrt{3}\sin x - \cos x) = \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\sin 30^\circ = \frac{1}{2}$ $\quad \sin\left[x - \frac{\pi}{3}\right] = \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$ $\quad \Rightarrow \sin\left[x - \frac{\pi}{3}\right]$

⑥ $\sin\left[x + \frac{\pi}{6}\right] = \cos x \quad ; \quad \tan x = \frac{1}{\sqrt{3}}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin x\cos\left[\frac{\pi}{6}\right] + \cos x\sin\left[\frac{\pi}{6}\right] = \cos x$

$\Rightarrow \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = 1\cos x$

$\frac{\sqrt{3}}{\cancel{2}}\sin x = \frac{1}{\cancel{2}}\cos x \Rightarrow \frac{\sin x}{\cos x} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan x = \frac{1}{\sqrt{3}}$

⑦ $2\sin\left(x + \frac{5\pi}{6}\right) + 2\sin\left(x + \frac{10\pi}{6}\right) = (1-\sqrt{3})\sin x + (1-\sqrt{3})\cos x$

$\Rightarrow \sin(A+B) = \sin A\cos B + \cos A\sin B.$

$$2\left[\sin x\cos\left[\frac{5\pi}{6}\right] + \cos x\sin\left[\frac{5\pi}{6}\right]\right] + 2\left[\sin x\cos\left(\frac{10\pi}{6}\right) + \cos x\sin\left[\frac{10\pi}{6}\right]\right]$$

$$\Rightarrow 2\left[(\sin x)\left[-\frac{\sqrt{3}}{2}\right] + \frac{1}{2}\cos x\right] + 2\left(\sin x\left(\frac{1}{2}\right) + \cos x\left(-\frac{\sqrt{3}}{2}\right)\right)$$

$$\cancel{2\left[\frac{1}{2}\right]\left[1 +\right.}$$

$$2\left[\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x\right] + 2\left[\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right]$$

$$\cancel{2}\left[\frac{1}{\cancel{2}}\right](\cos x - \sqrt{3}\sin x) + \cancel{2}\left(\frac{1}{\cancel{2}}\right)(\sin x - \sqrt{3}\cos x)$$

$$\Rightarrow \cos x - \sqrt{3}\sin x + \sin x - \sqrt{3}\cos x$$

$$\Rightarrow \cos x - \sqrt{3}\cos x + \sin x - \sqrt{3}\sin x$$

$$\Rightarrow \cos x(1-\sqrt{3}) + \sin(1-\sqrt{3})$$

$\therefore$ LHS = RHS

BF MATHS