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8.1 Parametric Equations

1a) $x = t - 3$ — ③ $y = t^2 - 1$ — ②

$x + 3 = t$ — ①

Sub ① in ②

$y = (x+3)^2 - 1$

Domain $= -6 \leq x \leq 0$

Range $= -1 \leq y \leq 8$

b) $x = \frac{2}{t}$ — ① $y = t^2 + 1$ — ②

$t = \frac{2}{x}$ — ③

Sub ③ in ②

$y = \left(\frac{2}{x}\right)^2 + 1$ Domain $= x > 0$

$y = \frac{4}{x^2} + 1$ Range $= y > 1$

c) $x = 2t - 1$ — ① $y = \frac{3}{t^2}$ — ②

$\frac{x+1}{2} = t$ — ③

Sub ③ in ②

$y = \frac{3}{\left(\frac{x+1}{2}\right)^2}$

Domain: $x \geqslant 1$

$y = \frac{12}{(x+1)^2}$

Range: $0 < y \leq 3$

8.1 Parametric Equations

2a) $x = \ln(t+2)$ — ① $y = \dfrac{1}{t+3}$ — ②

$t+3 = \dfrac{1}{y}$

$t = \dfrac{1}{y} - 3$ — ③

Sub ③ in ①

$x = \ln\left(\dfrac{1}{y} - 3 + 2\right)$

$e^x = \dfrac{1}{y} - 1$

$e^x + 1 = \dfrac{1}{y} \implies y = \dfrac{1}{e^x + 1}$

Domain : $x > 0$ Range : $0 < y < \dfrac{1}{2}$

b) $x = \ln(4+t)$ — ① $y = t^2 + 10t + 25$ — ③

$e^x = 4 + t$

$e^x - 4 = t$ — ②

Sub ② in ③

$y = (e^x - 4)^2 + 10(e^x - 4) + 25$

$y = e^{2x} - 8e^x + 16 + 10e^x - 40 + 25$

$y = e^{2x} + 2e^x + 1$

Domain : $x > 0$ Range : $y > 4$

c) $x = e^{2t}$ $y = e^{6t} + 2e^{4t}$

$y = (e^{2t})^3 + 2(e^{2t})^2$

$y = x^3 + 2x^2$

Domain : $x > 0$, Range : $y > 0$

8.1 Parametric equations

3a i) $x = t^{1/2} + 1$ — ① $y = 2t^{1/2} + 7$ — ③

$x - 1 = t^{1/2}$ — ②

Sub ② in ③

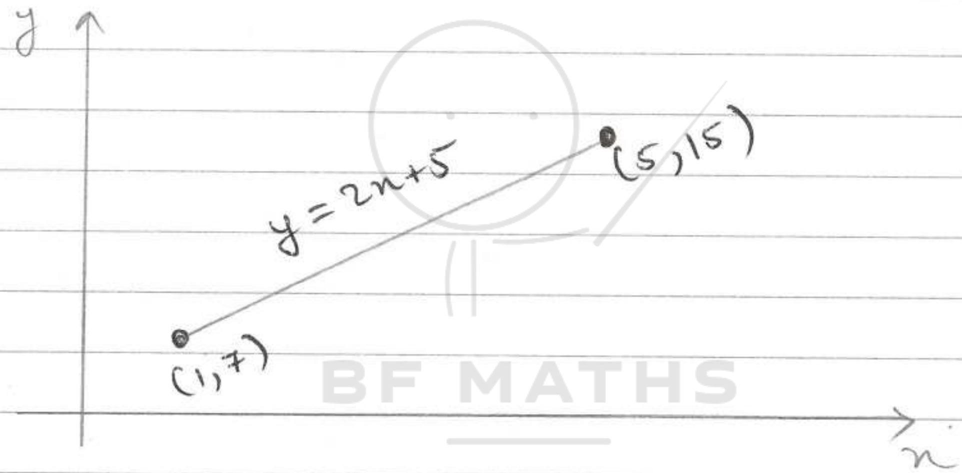
$y = 2(x-1) + 7$

$y = 2x + 5$

ii) Domain : $1 < x \leq 5$

Range : $7 < y \leq 15$

iii)



b i) $x = 2t + 1$ — ① $y = 4t^2 + 14t + 12$ — ③

$\frac{x-1}{2} = t$ — ②

Sub ② in ③

$y = 4\left(\frac{x-1}{2}\right)^2 + 14\left(\frac{x-1}{2}\right) + 12$

$y = \cancel{4}\left(\frac{x^2 - 2x + 1}{\cancel{4}}\right) + 7x - 7 + 12$

$y = x^2 - 2x + 1 + 7x - 7 + 12$

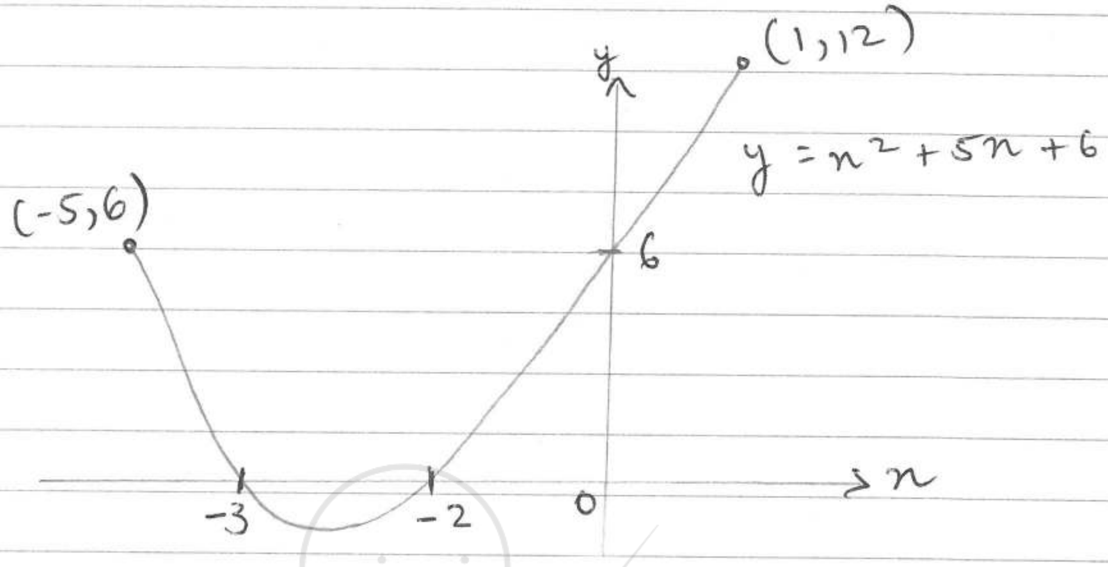
$y = x^2 + 5x + 6$

8.1 Parametric Equations

b ii] $-5 \le x \le 1$: Domain

$-\frac{1}{4} \le y \le 12$: Range

iii]



c i] $x = \ln(2-t)$ — ③ $y = 5 - t$ — ②

$t = 5 - y$ — ①

Sub ① into ③

$x = \ln(2 - 5 + y)$

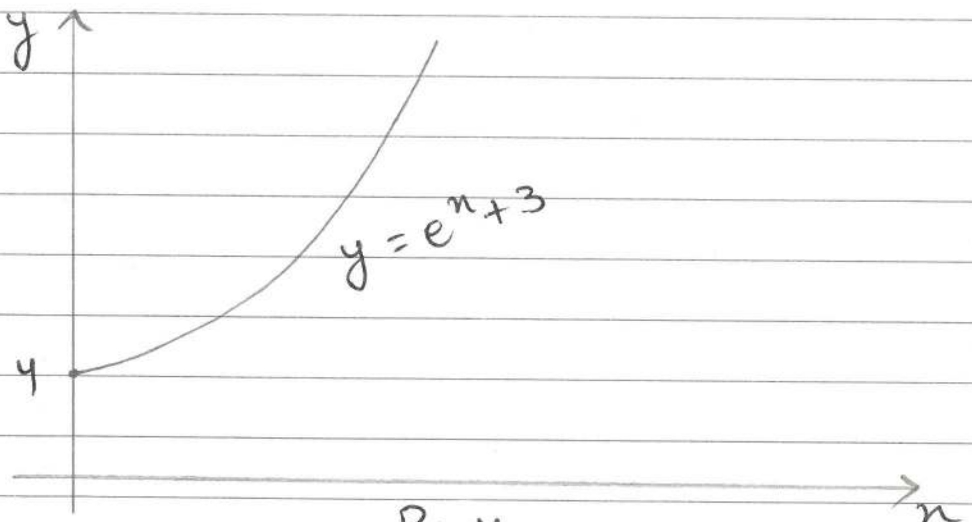
$e^x = -3 + y$

$y = e^x + 3$

ii] Domain : $x > 0$

Range : $y > 4$

iii]



8.1 Parametric Equations

4a) $x = 3t$ — ① $y = (1-t)(9-t)$ — ③

$t = \frac{x}{3}$ — ②

Sub ② in ③

$$y = \left(1 - \frac{x}{3}\right)\left(9 - \frac{x}{3}\right)$$

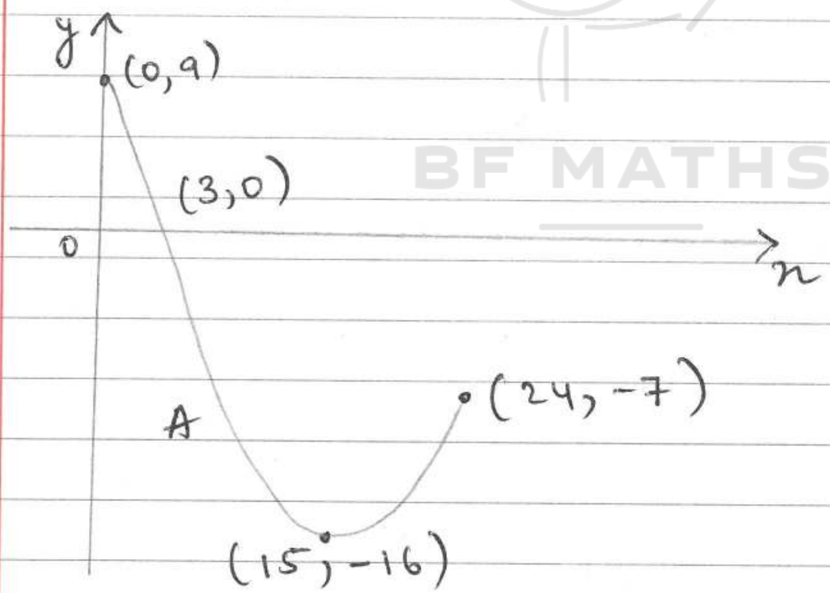
$$y = \left(9 - \frac{x}{3} - 3x + \frac{x^2}{9}\right)$$

$$y = \left(9 - \frac{10}{3}x + \frac{x^2}{9}\right)$$

Domain : $0 \le x \le 24$

Range : $-16 \le y \le 9$

b)



5a) $x = \ln(t+5)$ — ① $y = 2t + 12$ — ②

$\frac{y-12}{2} = t$ — ③

Sub ③ in ①

$$x = \ln\left(\frac{y-12}{2} + 5\right)$$

8.1 Parametric Equations

5a) Cont.

$e^x = \frac{y-12}{2} + 5$

$2e^x = y - 12 + 10$

$2e^x = y - 2$

$y = 2e^x + 2$

5b) When $x = 0$

$y = 2(1) + 2$

$y = 4$

Minimum value is $(0, 4)$

6a) $x = e^{3t}$ — ① $y = e^{9t} - 2e^{6t} - 5e^{3t} + 6$ — ②

$y = (e^{3t})^3 - 2(e^{3t})^2 - 5e^{3t} + 6$ — ③

Sub ① in ③

$y = x^3 - 2x^2 - 5x + 6$

Sub $x = -2$ in $f(x)$

$f(x) = x^3 - 2x^2 - 5x + 6$

$f(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$

$= 0$

Hence $(x+2)$ is a factor of $f(x)$

8.1 Parametric Equations

6a) Cont.

$$
\begin{array}{r|l}
 & x^2 - 4x + 3 \\
\hline
x+2 & x^3 - 2x^2 - 5x + 6 \\
 & x^3 + 2x^2 \\
 & - \quad - \\
\hline
 & -4x^2 - 5x + 6 \\
 & -4x^2 - 8x \\
 & + \\
\hline
 & 3x + 6 \\
 & 3x + 6 \\
\hline
 & 0
\end{array}
$$

$y = (x+2)(x^2 - 4x + 3)$

$y = (x+2)(x-3)(x-1)$

$y = (x-1)(x+2)(x-3)$

b) when $y = 0$

$x = 1$, $x = -2$, $x = 3$

$x = (1, 0)$, $(3, 0)$

when $x = 0$

$y = 6$

$y = (0, 6)$

So : x-axis $= (1,0)$ & $(3,0)$

y-axis $= (0, 6)$

8.1 Parametric Equations

7a) $C_1 : x = 5t^2$ — ① $y = 3t^2 + 2$ — ②

$\frac{x}{5} = t^2$ — ③

Sub ③ in ②

$y = 3\left(\frac{x}{5}\right) + 2$

$y = \frac{3x}{5} + 2$

$C_2 : x = 3\sqrt{t} - 3$ — ① $y = 9 - 5\sqrt{t}$ — ③

$\frac{x+3}{3} = \sqrt{t}$ — ②

Sub ② in ③

$y = 9 - 5\left(\frac{x+3}{3}\right)$

$y = 9 - \left(\frac{5x + 15}{3}\right)$

$3y = 27 - 5x - 15$

$y = -\frac{5x}{3} + 4$

$\frac{3}{5} \times -\frac{5}{3} = -1$

$\therefore$ The line segments are perpendicular

7b) $C_1 = (0, 2)$ $(20, 14)$

$= \sqrt{12^2 + 20^2}$

$= 4\sqrt{34}$

8.1 Parametric Equations

7b] Cont.

$C_2 \Rightarrow (-3, 9) \quad (3, -1)$

$= \sqrt{10^2 + (-6)^2}$

$= 2\sqrt{36}$

8] $x = 4t^2$ —— ① $\quad y = 8t^3$ —— ②

$\sqrt{\frac{x}{4}} = t$ —— ③

Sub ③ in ②

$y = 8\left(\sqrt{\frac{x}{4}}\right)^3$

$y^2 = 8^2\left(\sqrt{\frac{x}{4}}\right)^{3 \times 2}$

$y^2 = 64\left(\frac{x^3}{64}\right)$

$y^2 = x^3$