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7·5 : Simplifying $a\cos x \pm b\sin x$

① a) $20\sin\theta + 21\cos\theta \equiv R\sin(\theta + a)$

$\Rightarrow R = \sqrt{20^2+21^2} = 29$ $\Rightarrow \tan a = \frac{21}{20}$

because $R\sin(\theta + a)$ the coefficient of sin has to be the denominator.

b) $\sqrt{3}\cos\theta - \sqrt{5}\sin\theta = R\cos(\theta + a)$

$\Rightarrow R = \sqrt{(\sqrt{3})^2+(\sqrt{5})^2} = \sqrt{8} = 2\sqrt{2}$ $\Rightarrow \tan a = \frac{\sqrt{5}}{\sqrt{3}}$

c) $2\sin\theta - 3\cos\theta \equiv R\sin(\theta - a)$

$\Rightarrow \sqrt{2^2+3^2} = \sqrt{13} = R$ $\Rightarrow \tan a = \frac{3}{2}$

② $R > 0$; $0 < a < 90°$

a) $2\sin\theta + 1\cos\theta = R\cos(\theta - a)$

$R = \sqrt{2^2+1^2} \Rightarrow \sqrt{5}$ $\tan a = 2$ $a = \arctan(2)$

$a = 63.4°$ (3sf)

b) $12\cos\theta - 5\sin\theta \equiv R\cos(\theta + a)$

$R = \sqrt{12^2+5^2} = 13$ $\tan a = \frac{5}{12}$ $a = \arctan\left[\frac{5}{12}\right]$

$a = 22.6°$ (3sf)

c) $4\sin\theta - 2\cos\theta \equiv R\sin(\theta - a)$

$R = \sqrt{4^2+2^2} = 2\sqrt{5}$ $\tan a = \frac{2}{4}$ $a = \arctan(0.5)$

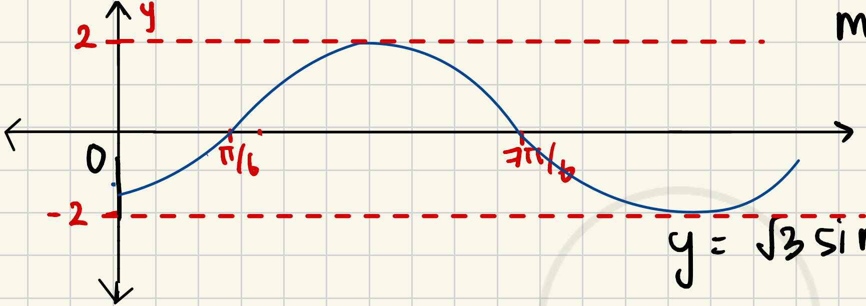
$a = 26.6°$ (3sf)

③ a) i) $\sqrt{3}\sin\theta - 1\cos\theta = R\sin(\theta - a)$

$R = \sqrt{(\sqrt{3})^2+(1)^2} = 2$ $\tan a = \frac{1}{\sqrt{3}}$ $a = \frac{\pi}{6}$

$\sqrt{3}\sin\theta - \cos\theta \Rightarrow 2\sin\left(\theta - \frac{\pi}{6}\right)$

iii)



move right $\begin{pmatrix} \pi/6 \\ 0 \end{pmatrix}$

Stretch $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

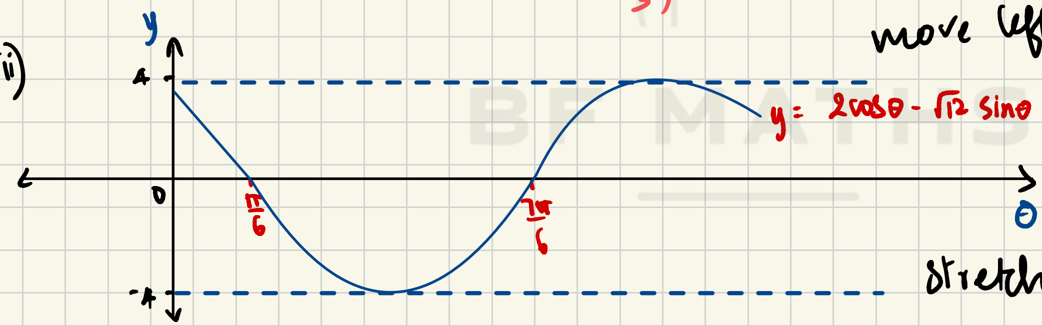
$y = \sqrt{3}\sin\theta - \cos\theta$

b) i) $2\cos\theta - \sqrt{12}\sin\theta \qquad R\cos(\theta + a)$

$R = \sqrt{2^2 + \sqrt{12}^2} = 4 \qquad \tan a = \frac{\sqrt{12}}{2} \qquad a = \frac{\pi}{3}$

$2\cos\theta - \sqrt{12}\sin\theta = 4\cos\left(\theta + \frac{\pi}{3}\right)$

ii)



move left by $\begin{pmatrix} \pi/3 \\ 0 \end{pmatrix}$

Stretch by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

c) $2\sin\theta + 2\cos\theta \qquad R\sin(\theta + a)$

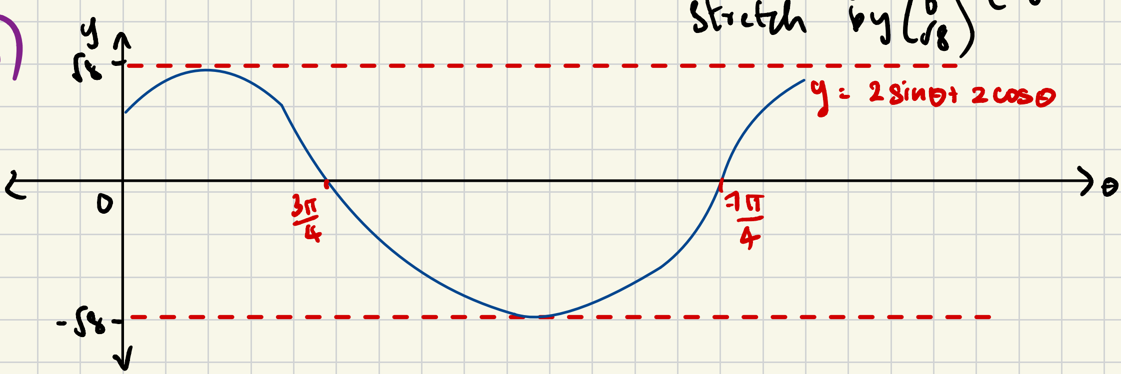
$R = \sqrt{2^2 + 2^2} = 2\sqrt{2} = \sqrt{8} \qquad \tan a = \frac{2}{2} \qquad a = \frac{\pi}{4}$

i) $2\sin\theta + 2\cos\theta = 2\sqrt{2}\sin(\theta + \pi/4)$

move left by $\begin{pmatrix} -\frac{\pi}{4} \\ 0 \end{pmatrix}$

Stretch by $\begin{pmatrix} 0 \\ \sqrt{8} \end{pmatrix}$

ii)



④ $h(\theta) = 2\sin\theta + 4\cos\theta$

a) $R = \sqrt{2^2+4^2} = 2\sqrt{5}$ $\tan a = \frac{4}{2} = 63.43°$

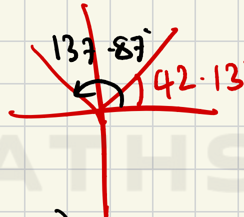
$\Rightarrow 2\sqrt{5}\sin(\theta + 63.43°)$

b) $2\sin\theta + 4\cos\theta = 3$

$\Rightarrow 2\sqrt{5}\sin(\theta + 63.43°) = 3$

$\Rightarrow \sin(\theta + 63.43°) = \frac{3}{2\sqrt{5}}$ $\theta + 63.43° \ \sin^{-1}\left[\frac{3}{2\sqrt{5}}\right]$

$\theta + 63.43° = \cancel{42.13}, 137.87°, 402.13$



$\theta = 74.4°, 338.7°$ (1dp)

⑤ $3\cos 2\theta + 2\sin 2\theta = R\cos(2\theta - a)$

a) $R = \sqrt{3^2+2^2} = \sqrt{13}$ $\tan a = \frac{2}{3}$ $a = 0.588$

$= \sqrt{13}\cos(2\theta - 0.588)$

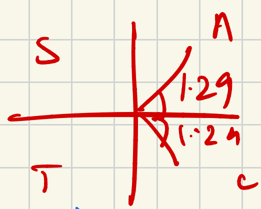
b) $3\cos 2\theta + 2\sin 2\theta = 1$

$\sqrt{13}\cos(2\theta - 0.588) = 1$ $\cos(2\theta - 0.588) = \frac{1}{\sqrt{13}}$

$\Rightarrow 2\theta - 0.588 = \cos^{-1}\left[\frac{1}{\sqrt{13}}\right]$

$-360° \le x \le 360°$

$\Rightarrow 2\theta - 0.588 = 1.29, -1.29, 4.99, -4.99$



$\theta = -2.20, -0.35, 0.94, 2.79$ (2dp)

c) $\sqrt{13}\cos(\theta - 0.588) = k$

$k > \sqrt{13}$, $k < -\sqrt{13}$; So the value of cos exceeds the value 1 and not in the range of cos $(-1 \le y \le 1)$. Therefore no solution.

(6) $8\sin\theta - 15\cos\theta = R\sin(\theta - a)$

$R = \sqrt{8^2 + 15^2} = 17$ $\quad \tan a = \dfrac{15}{8}$ $\quad a = 1.08$

$\Rightarrow 17\sin(\theta - 1.08)$

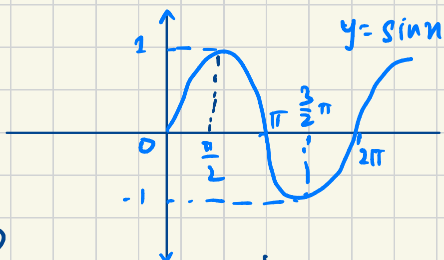
b) i) $g(\theta) = \dfrac{10}{23 + 17\sin(\theta - 1.08)} = \dfrac{10}{23 - 17} = \dfrac{10}{6} = \dfrac{5}{3}$

$\sin(\theta - 1.08) = -1$

ii) $\sin(\theta - 1.08) = -1$ $\quad \theta - 1.08 = \arcsin(-1)$

$\theta - 1.08 = \dfrac{3}{2}\pi$

$\theta = \dfrac{3}{2}\pi + 1.08 = 5.79$ (2dp)



c) i) $g(\theta) = \dfrac{10}{23 + 17\sin(\theta - 1.08)} = \dfrac{10}{23 + 17} = \dfrac{10}{40} = \dfrac{1}{4}$

$\sin(\theta - 1.08) = 1$

ii) $\sin(\theta - 1.08) = 1$ $\quad \theta - 1.08 = \arcsin(1)$

$\theta - 1.08 = \dfrac{\pi}{2}$ $\quad \theta = 2.65$ (2dp)

7 a) $2\sin\left(x-\frac{\pi}{6}\right)-(\sqrt{3}-2)\sin x = 2\sin x-\cos x$

$2\left[\sin(A-B)\right]-\sqrt{3}\sin x+2\sin x$

$= 2\left(\sin x\cos\frac{\pi}{6}-\cos x\sin\frac{\pi}{6}\right)-\sqrt{3}\sin x+2\sin x$

$\Rightarrow \sqrt{3}\cancel{\sin x}-\cos x-\sqrt{3}\cancel{\sin x}+2\sin x$

$= 2\sin x-\cos x$

b) $2\sin\left(x-\frac{\pi}{6}\right)-(\sqrt{3}-2)\sin x = \frac{1}{2}$

$\Rightarrow 2\sin x-\cos x = \frac{1}{2} \quad \Rightarrow \left(\frac{1}{2}+\cos x\right)^2=(2\sin x)^2$

$\Rightarrow \frac{1}{4}+\cos^2 x+\cos x-4\sin^2 x = 0$

$\Rightarrow \frac{1}{4}+\cancel{\cos^2 x}+\cos x-4+4\cancel{\cos^2 x} = 0$

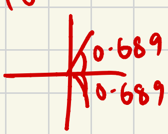
$\Rightarrow 5\cos^2 x+\cos x-\frac{15}{4} = 0$

$-\pi \le x \le 2\pi$

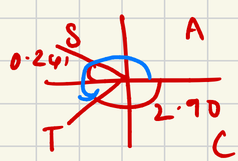
$\Rightarrow 20\cos^2 x+4\cos x-15=0 \qquad -180^\circ \le x \le 360^\circ$

$\cos x = \frac{-1+2\sqrt{19}}{10} \qquad \cos x = \frac{-1-2\sqrt{19}}{10}$

$x = 0.689,$



$x = -2.90$



$x = \pi+0.241$

$x = 3.38$

$\Rightarrow x = -2.90,\ 0.69,\ 3.38$

c) $\left(2\sin\left(x-\frac{\pi}{6}\right)-(\sqrt{3}-2)\sin x\right)^4$

$2\sin x - \cos x = 0 \Rightarrow R\cos(x+a)$

$\Rightarrow R = \sqrt{2^2+1^2} = \sqrt{5}\cos(x+1.107)$

$\Rightarrow \cos(x+1.107) = 1 \Rightarrow (\sqrt{5})^4 = 25$