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6·4 Trigonometric identities

1a] $\operatorname{cosec}^2 x + \tan^2 x \equiv \sec^2 x + \cot^2 x$

Using LHS $= \operatorname{cosec}^2 x + \tan^2 x$

$= (1 + \cot^2 x) + (\sec^2 x - 1)$

$= \cot^2 x + \sec^2 x$

LHS = RHS

b] $\cot^2 x + \cos^2 x \equiv (\operatorname{cosec} x - \sin x)(\operatorname{cosec} x + \sin x)$

Using LHS $= \cot^2 x + \cos^2 x$

$= (\operatorname{cosec}^2 x - 1) + (1 - \sin^2 x)$

$= \operatorname{cosec}^2 x - \sin^2 x$

$= (\operatorname{cosec} x - \sin x)(\operatorname{cosec} x + \sin x)$

c] $\dfrac{1}{1+\sin x} + \dfrac{1}{1-\sin x} \equiv 2 + 2\tan^2 x$

LHS $= \dfrac{1}{1+\sin x} + \dfrac{1}{1-\sin x}$

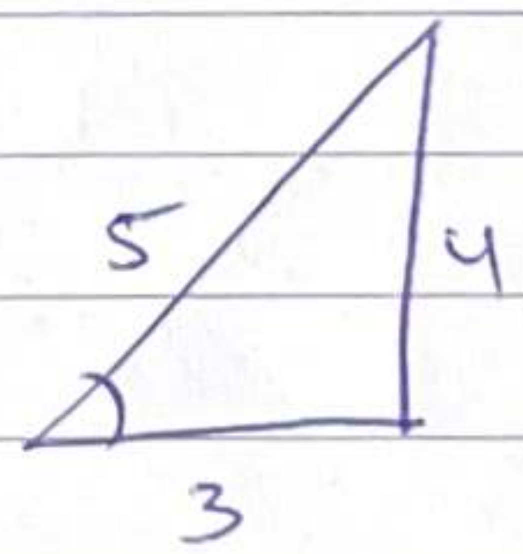
$= \dfrac{1 - \sin x + 1 + \sin x}{(1+\sin x)(1-\sin x)}$

$= \dfrac{2}{1-\sin^2 x}$

$= \dfrac{2}{\cos^2 x} \Rightarrow 2\sec^2 x \Rightarrow 2(1+\tan^2 x)$

$\Rightarrow 2 + 2\tan^2 x$ Hence proved.

2a] $\cos x = -\dfrac{3}{5}$



As cos is -ve in the 2nd quadrant.

2b] $\cot x = \dfrac{1}{\tan x} = \dfrac{1}{-\frac{4}{3}}$

$\cot x = -\dfrac{3}{4}$

As cot x is -ve in the 2nd quadrant

6.4 Trigonometric identities

2c] $\text{Cosec}\, x = \dfrac{1}{\sin x} \Rightarrow \dfrac{1}{\frac{4}{5}} = \dfrac{5}{4}$

3a] $\sec^2 x + \tan x = 1 \qquad -\pi \le x \le \pi$

$(1 + \tan^2 x) + \tan x = 1$

$\tan^2 x + \tan x = 0$

$\tan x(\tan x + 1) = 0$

$\tan x = 0 \quad , \quad \tan x = -1$

$x = 0 \quad , \quad x = -\dfrac{1}{4}\pi$

So $x = -\pi, -\dfrac{\pi}{4}, 0, \dfrac{3\pi}{4}, \pi.$

b] $3\cot^2 x + 9\,\text{Cosec}\, x + 1 = 0$

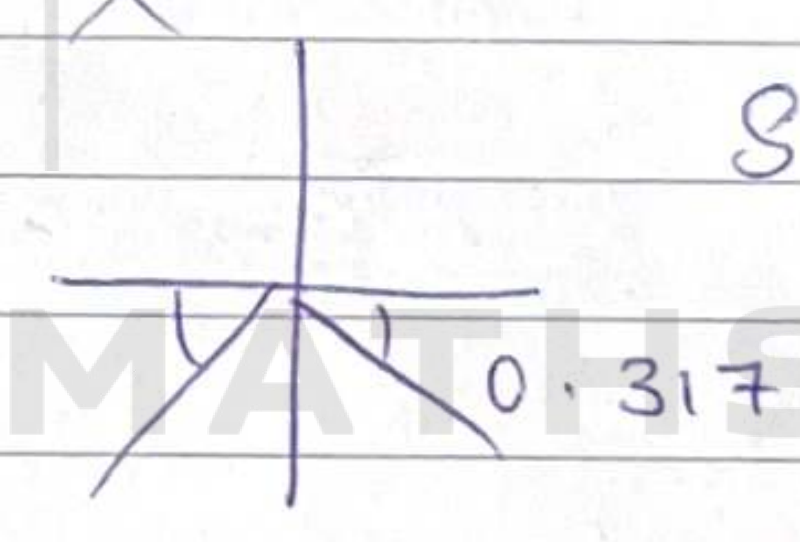
$3(\text{Cosec}^2 x - 1) + 9\,\text{Cosec}\, x + 1 = 0$

$3\,\text{Cosec}^2 x + 9\,\text{Cosec}\, x - 2 = 0$

$\text{Cosec}\, x = \dfrac{-9 + \sqrt{105}}{6}$ X , $\text{Cosec}\, x = \dfrac{-9 - \sqrt{105}}{6}$

$\sin x = \dfrac{6}{-9 - \sqrt{105}}$

$x = 3.46, 5.97$
(3sf)



c] $\text{Cosec}^2 x + \cot^2 x = 3$

$\text{Cosec}^2 x + (\text{Cosec}^2 x - 1) = 3$

$2\,\text{Cosec}^2 x = 4$

$\text{Cosec}^2 x = 2$

$\text{Cosec}\, x = \sqrt{2}$

$x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

6.4 Trigonometric identities

4] $7\cot^2 x - 5\operatorname{cosec}^2 x = 2$

$7\left(\frac{\cos^2 x}{\sin^2 x}\right) - 5\left(\frac{1}{\sin^2 x}\right) = 2$

$7\cos^2 x - 5 = 2\sin^2 x$

$7\cos^2 x - 5 = 2(1-\cos^2 x)$

$7\cos^2 x - 5 = 2 - 2\cos^2 x$

$9\cos^2 x = 7$

$\cos^2 x = \frac{7}{9}$

$\cos x = \sqrt{\frac{7}{9}} \implies \cos x = \pm\frac{\sqrt{7}}{3}$

Since $\cos x$ is obtuse, $\cos x = -\frac{\sqrt{7}}{3}$

5] $\sec^2 x = 5 + 3\tan x$

$1 + \tan^2 x = 5 + 3\tan x$

$\tan^2 x - 3\tan x - 4 = 0$

$\tan x = 4$ $\quad\quad$ $\tan x = -1$

$x = 76.0$ $\quad\quad$ $x = -45$

$x = 76.0°, 256°, 135°, 315°.$

6a] $\cot^2 p = -4\operatorname{cosec} p$

$\operatorname{cosec}^2 p - 1 = -4\operatorname{cosec} p$

$\operatorname{cosec}^2 p + 4\operatorname{cosec} p - 1 = 0$

$\operatorname{cosec} p = -2+\sqrt{5}$ ✗ , $\operatorname{cosec} p = -2-\sqrt{5}$

b] $\sin p = \frac{1}{-2-\sqrt{5}}$

$\sin p = \frac{1}{-2-\sqrt{5}} \times \frac{-2+\sqrt{5}}{-2+\sqrt{5}}$

6.4 Trigonometric identities

6b] Cont.

$\sin p = -\frac{2+\sqrt{5}}{4-\sqrt{5}} \Longrightarrow \sin p = -\frac{2+\sqrt{5}}{-1}$

$\sin p = 2-\sqrt{5}$ Hence, proved.

c] $\sin p = 2-\sqrt{5}$

$p = 0.24$

So $p = 6.0, 3.4$.

7] $2\operatorname{cosec}^2 x + 5\cot x = 5$

$2(1+\cot^2 x) + 5\cot x - 5 = 0$

$2\cot^2 x + 5\cot x - 3 = 0$

$\cot x = \frac{1}{2}$, $\cot x = -3$

$\tan x = 2$ $\tan x = -\frac{1}{3}$

$x = 1.11, 2.82$

$x = -0.32, -2.03$

So $x = -2.03, -0.32, 1.11, 2.82$.