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3.7: Recurrence Relations

1 a) Given: $u_{n+1} = 2u_n - 1$, $u_1 = 3$

$u_1 = 3$

$u_2 = 2(3) - 1 = 5$

$u_3 = 2(5) - 1 = 9$

$u_4 = 2(9) - 1 = 17$

$\Rightarrow 3, 5, 9, 17$

b) Given: $u_{n+1} = u_n - 4$, $u_1 = 10$

$u_1 = 10$

$u_2 = 10 - 4 = 6$

$u_3 = 6 - 4 = 2$

$u_4 = 2 - 4 = -2$

$\Rightarrow 10, 6, 2, -2$

c) Given: $u_{n+1} = (u_n)^2/2$, $u_n = 4$

$u_1 = 4$

$u_2 = (4)^2/2 = 8$

$u_3 = (8)^2/2 = 32$

$u_4 = (32)^2/2 = 512$

$\Rightarrow 4, 8, 32, 512$

2 a) Given: 3, 7, 11, 15, 19 i) it is increasing, $u_{n+1} > u_n$

b) Given: 1, 0, -1, 0, 1, 0, -1, ... i) it is periodic, $u_{n+k} = u_n$ ii) 4

c) Given: 11, 8, 5, 2, -1 ... i) it is decreasing, $u_{n+1} < u_n$

3 a) Given: $u_n = 3n - 1$ i) 2, 5, 8, 11, 14 ii) increasing

b) Given: $u_n = 2^{1-n}$ i) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ ii) decreasing

c) Given: $u_n = \sin(180n - 30)$ i) $\frac{1}{2}, \frac{-1}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{1}{2}$ ii) periodic iii) 2

d) Given: $u_{n+1} = 2u_n + 1$, $u_1 = 1$ i) 1, 3, 7, 15, 31 ii) increasing

e) Given: $u_{n+1} = 4 - u_n$, $u_1 = 8$ i) 8, -4, 8, -4, 8 ii) periodic iii) 2

f) Given: $u_{n+1} = u_n - 4$, $u_1 = 10$ i) 10, 6, 2, -2, -6 ii) decreasing

4 Given: $a_1 = k$ $\quad a_{n+1} = a_n + (-1)^n$

a) $a_2 = k + (-1)^1 = k - 1$ $\quad a_3 = k - 1 + (-1)^2 = k \cancel{-1} \cancel{+1} = k$

$a_4 = k + (-1)^3 = k - 1$ $\quad a_2 = a_4 = k - 1$

b) Given: $k = 3$ $\Rightarrow$ 3, 2, 3, 2 ....

$$\sum_{r=1}^{400} a_r = 3(200) + 2(200) = 1000$$

c) $a_{399} = 3$ (as 399 is a odd)

5. Given: $u_{n+1} = pu_n + q \quad u_1 = 4 \quad u_2 = 3 \quad u_3 = 1$

$u_2 = p(4) + q = 4p + q$

$u_3 = p(4p+q) + q = 4p^2 + qp + q \Rightarrow 4p^2 + q(1+p)$

$4p + q = 3 \quad -①$

$\boxed{q = 3 - 4p}$

$4p^2 + q(1+p) = 1$

$\Rightarrow 4p^2 + (3 - 4p)(1+p) = 1$

$\Rightarrow \cancel{4p^2} + 3 + 3p - 4p - \cancel{4p^2} = 0$

$\Rightarrow -p + 2 = 0$

$\boxed{p = 2}$

$q = 3 - 4(2) = -5 \quad \boxed{q = -5}$

6. Given: $u_1 = 5 \quad u_{n+1} = ku_n - 8$

a) $u_2 = 5k - 8 \qquad u_3 = k(5k-8) - 8 = 5k^2 - 8k - 8$

b) $u_3 = 40 \Rightarrow 40 = 5k^2 - 8k - 8 \Rightarrow 5k^2 - 8k - 48 = 0$

$(k-4)(5k+12) = 0 \qquad k = 4$ or $k = -12/5$

$k = 4, \; k > 0$

c) $u_3 = 40 \qquad u_4 = 4(40) - 8 = 152 \qquad u_5 = 4(152) - 8 = 600$

7. Given: $u_n = \sin(90n^\circ), \; n \geq 1$

a) $1, 0, -1, 0, 1, 0 \ldots$ Order of sequence is 4

b) $$\sum_{r=1}^{200} u_r = 50(1) + 50(-1) + 100(0) = 50 - 50 = 0$$

8. Given: $a_1 = k \quad a_{n+1} = 5a_n + 4$

a) $a_2 = 5k + 4 \qquad a_3 = 5(5k+4) + 4 = 25k + 24$

b) $a_4 = 5(25k + 24) + 4 = 125k + 124$

$$\sum_{r=1}^{4} a_r = \underbrace{k}_{a_1} + \underbrace{5k+4}_{a_2} + \underbrace{25k+24}_{a_3} + \underbrace{125k+124}_{a_4} = 156k + 152 = 4(39k + 38)$$, so it is a multiple of 4.

9. Given: $a_1 = 3 \quad a_{n+1} = 1 - \dfrac{1}{a_n - 2}$

a) $\sum_{r=1}^{50} a_r \Rightarrow 3, 0, \frac{3}{2}, 3, \ldots$ (periodic (3)) $\Rightarrow 16(3) + 16(0) + 16(3/2) + 3 + 0 = 75$

b) $\sum_{r=1}^{50} a_r + \sum_{r=1}^{49} a_r \Rightarrow \sum_{r=1}^{50} a_r = 75 \quad \sum_{r=1}^{49} a_r = 75 \; (48+24+3) \Rightarrow 75 + 75 = 150$