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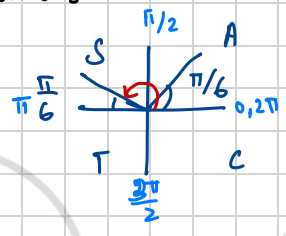
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5.4 Solving trigonometric equations

① $0 \leq \theta \leq 2\pi$

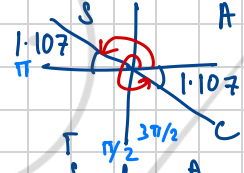
a) $\sin \theta = 0.5 \Rightarrow \theta = \arcsin(0.5) \Rightarrow \theta = \frac{\pi}{6}$

$\Rightarrow \pi - \frac{\pi}{6} = \frac{5\pi}{6} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$



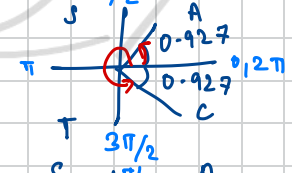
b) $\tan \theta = -2 \Rightarrow \theta = \arctan(-2) \Rightarrow \theta = -1.107$

$\Rightarrow \pi - 1.107, 2\pi - 1.107 \Rightarrow \theta = 2.03, 5.18$



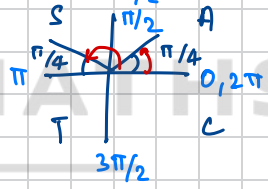
c) $5 \cos \theta = 3 \Rightarrow \theta = \arccos\left(\frac{3}{5}\right) \Rightarrow \theta = 0.927$

$\Rightarrow 2\pi - 0.927 \Rightarrow \theta = 0.927, 5.36$



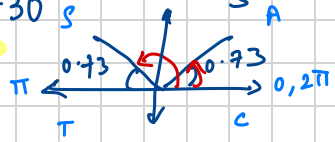
d) $\sqrt{2} \sin \theta + 1 = 2 \Rightarrow \theta = \arcsin\left[\frac{1}{\sqrt{2}}\right] \Rightarrow \theta = \frac{\pi}{4}$

$\Rightarrow \pi - \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}$



② a) $2 + 3 \sin \theta = 4, 0 \leq \theta \leq \pi \Rightarrow 3 \sin \theta = 2 \Rightarrow \sin \theta = \frac{2}{3}$

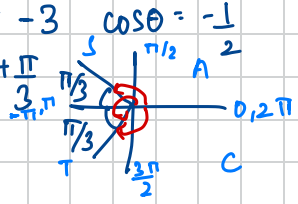
$\theta = \arcsin\left(\frac{2}{3}\right) \Rightarrow \theta = 0.730, \pi - 0.730$
 $\Rightarrow \theta = 0.730, 2.41$ (3SF)



b) $6 \cos \theta - 1 = -4, -\pi \leq \theta \leq 2\pi \Rightarrow 6 \cos \theta = -3 \Rightarrow \cos \theta = -\frac{1}{2}$

$\theta = \arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3} \Rightarrow \theta = \frac{2\pi}{3}, \frac{\pi + \pi}{3}, \frac{-\pi + \pi}{3}$

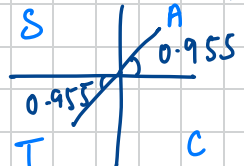
$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{-2\pi}{3}$



c) $\sqrt{2} \tan \theta - 7 = -5, -2\pi \leq \theta \leq \pi \Rightarrow \tan \theta = \frac{2}{\sqrt{2}} \Rightarrow \theta = \arctan\left(\frac{2}{\sqrt{2}}\right)$

$\Rightarrow \theta = 0.955, \theta = -2\pi + 0.955, -\pi + 0.955,$

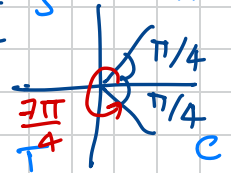
$\Rightarrow \theta = -5.33, -2.17, 0.955$ (3SF)



d) $\sqrt{2} \cos 2\theta = 0.6 = 0.4$, $\pi \leq \theta \leq 3\pi \Rightarrow \cos 2\theta = \frac{1}{\sqrt{2}}$ $2\pi \leq 2\theta \leq 6\pi$
S A

$2\theta = \frac{\pi}{4}, \frac{15\pi}{4}, \frac{17\pi}{4}, \frac{23\pi}{4}$

$\theta = \frac{9\pi}{8}, \frac{15\pi}{8}, \frac{17\pi}{8}, \frac{23\pi}{8}$



③ $0 \leq \theta \leq 2\pi$, a) $\tan^2 \theta + 3 \tan \theta = -2 = \tan^2 \theta + 3 \tan \theta + 2 = 0$

$(\tan \theta + 1)(\tan \theta + 2) = 0$

$\tan \theta = -1$ or $\tan \theta = -2$

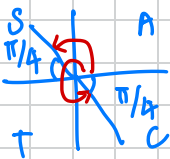
$\tan \theta = -1$

$\theta = \arctan(-1) \Rightarrow \theta = -\frac{\pi}{4}$

$\theta = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\theta = 2.03, 2.36, 5.18, 5.50$ (3sf)

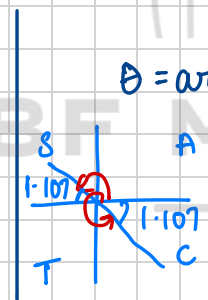


$\tan \theta = -2$

$\theta = \arctan(-2) \Rightarrow \theta = -1.107$

$\theta = \pi - 1.107, 2\pi - 1.107$

$\theta = 2.03, 5.18$



b) $3 \sin^2 \theta - 2 \cos \theta + 1 = 0$

$\sin^2 \theta = 1 - \cos^2 \theta$

$\Rightarrow 3(1 - \cos^2 \theta) - 2 \cos \theta + 1 = 0 \Rightarrow 3 - 3 \cos^2 \theta - 2 \cos \theta + 1 = 0$

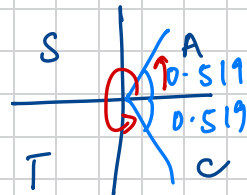
$3 \cos^2 \theta + 2 \cos \theta - 4 = 0 \Rightarrow \cos \theta = 0.8685$, ~~$\cos \theta = -1.535$~~

$-1 \leq \cos \theta \leq 1$

$\Rightarrow \theta = \arccos(0.8685) = 0.519$

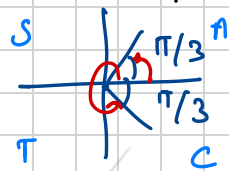
$\theta = 0.519, 2\pi - 0.519$

$\theta = 0.519, 5.76$ (3sf)



c) $\cos\left(\theta + \frac{\pi}{4}\right) = 0.5$ New Range $\Rightarrow \frac{\pi}{4} \leq \theta + \frac{\pi}{4} \leq \frac{9\pi}{4}$

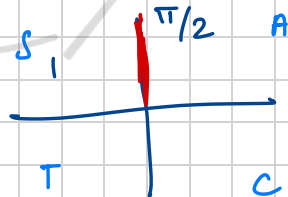
$\theta + \frac{\pi}{4} = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \theta + \frac{\pi}{4} = \frac{\pi}{3}$



$\theta + \frac{\pi}{4} = \frac{\pi}{3}, \frac{5\pi}{3} \Rightarrow \theta = \frac{\pi}{12}, \frac{17\pi}{12}$

d) $\sin(2\theta + 0.6) = 1$ New range $\Rightarrow 0 \leq 2\theta + 0.6 \leq 4\pi + 0.6$

$2\theta + 0.6 = \frac{\pi}{2}, \frac{5\pi}{2} \Rightarrow \theta = \left(\frac{\pi}{2} - 0.6\right) \div 2$



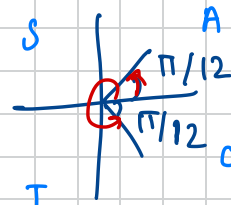
$\theta = \left(\frac{5\pi}{2} - 0.6\right) \div 2$

$\theta = 0.485, 3.63$ (3sf)

④ $0 \leq \theta \leq 2\pi$, a) $\cos\left(\theta - \frac{\pi}{18}\right) = \cos\left(\frac{\pi}{12}\right) \rightarrow \frac{\sqrt{6} + \sqrt{2}}{4}$

$\Rightarrow \cos\left(\theta - \frac{\pi}{18}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$ New Range; $-\frac{\pi}{18} \leq \theta - \frac{\pi}{18} \leq \frac{35\pi}{18}$

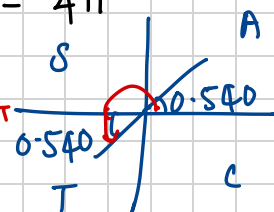
$\theta - \frac{\pi}{18} = \cos^{-1}\left[\frac{\sqrt{6} + \sqrt{2}}{4}\right] \Rightarrow \theta - \frac{\pi}{18} = \frac{\pi}{12}, \frac{23\pi}{12}$



$\Rightarrow \theta = \frac{\pi}{12} + \frac{\pi}{18}, \frac{23\pi}{12} + \frac{\pi}{18} = \frac{5\pi}{36}, \frac{71\pi}{36}$

b) $\tan 2\theta = 0.6$ New range: $0 \leq 2\theta \leq 4\pi$

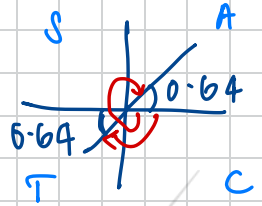
$2\theta = \tan^{-1}(0.6)$ $2\theta = 0.540, \pi + 0.540$
 $+ 2\pi \rightarrow 6.82, 9.96$ $+ 2\pi$



$2\theta = 0.540, 3.68, 6.82, 9.96$

$\theta = 0.270, 1.84, 3.41, 4.98$ (3sf)

$$5) a) 4 \sin x = 3 \cos x \Rightarrow \frac{\sin x}{\cos x} = \frac{3}{4} \quad \tan x = \frac{3}{4}$$



$$b) -2\pi \leq 2x \leq 2\pi \Rightarrow 2x = \tan^{-1}\left(\frac{3}{4}\right) = 0.64$$

$$2x = 0.64, \pi + 0.64, -2\pi + 0.64, -\pi + 0.64$$

$$2x = -5.64, -2.50, 0.64, 3.78$$

$$x = -2.8, -1.2, 0.3, 1.9 \quad (\text{idp})$$

$$6) 2 \cos^2 x + 3 \sin x = 3$$

$$\boxed{\cos^2 x = 1 - \sin^2 x}$$

$$a) 2(1 - \sin^2 x) + 3 \sin x = 3 \Rightarrow 2 - 2\sin^2 x + 3 \sin x - 3 = 0$$

$$\Rightarrow 2\sin^2 x - 3 \sin x + 1 = 0$$

$$b) 0 \leq x \leq \pi, (\sin x - 1)(\sin x - 0.5) = 0$$

$$\Rightarrow \sin x = 1 \Rightarrow \sin x = 0.5 \quad x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$

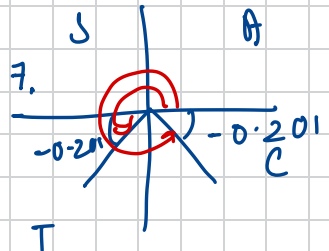
$$x = \frac{\pi}{2} \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$7) \sin\left(3\theta + \frac{\pi}{4}\right) = -0.2$$

$$0 \leq \theta \leq \frac{3\pi}{2}$$

$$\frac{\pi}{4} \leq 3\theta + \frac{\pi}{4} \leq \frac{19\pi}{4}$$

$$3\theta + \frac{\pi}{4} = \sin^{-1}(-0.2)$$



$$\Rightarrow 3\theta + \frac{\pi}{4} = 3.343, 6.082, 9.626, 12.37$$

$$\theta = 0.853, 1.77, 2.95, 3.86 \quad (\text{3sf})$$

$$b) 4 \cos x = 6 \tan x ; 0 \leq x \leq \pi \quad \boxed{1 - \sin^2 x}$$

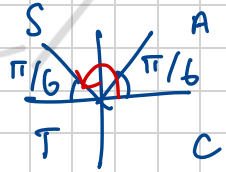
$$4 \cos x = 6 \frac{\sin x}{\cos x} \Rightarrow 4 \cos^2 x = 6 \sin x$$

$$= 4(1 - \sin^2 x) - 6 \sin x = 0 \Rightarrow 4 - 4 \sin^2 x - 6 \sin x = 0$$

$$\Rightarrow 4 \sin^2 x + 6 \sin x - 4 = 0 \quad (\sin x - 0.5)(\sin x + 2) = 0$$

$$\sin x = \frac{1}{2}$$

$$\cancel{\sin x = -2} \quad (-1 \leq \sin x \leq 1)$$



$$\hookrightarrow x = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}, \frac{5\pi}{6}$$

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