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4.2 Expanding $(a+bx)^n$

1a i] $\frac{1}{(2+3x)^2} \Rightarrow (2+3x)^{-2} \Rightarrow 2^{-2}\left(1+\frac{3}{2}x\right)^{-2}$

$\Rightarrow \frac{1}{4}\left(1+\frac{3}{2}x\right)^{-2}$

$= \frac{1}{4}\left(1 + (-2)\left(\frac{3}{2}x\right) + \frac{(-2)(-2-1)}{2!}\left(\frac{3}{2}x\right)^2 + \frac{(-2)(-2-1)(-2-2)}{3!}\left(\frac{3}{2}x\right)^3\right)$

$\Rightarrow \frac{1}{4}\left(1 - 3x + \frac{27}{4}x^2 - \frac{27}{2}x^3\right)$

$\Rightarrow \frac{1}{4} - \frac{3}{4}x + \frac{27}{16}x^2 - \frac{27}{8}x^3$

ii] $2+3x < 0$

$|x| < \frac{2}{3}$

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bi] $\sqrt{4-x} = (4-x)^{1/2} \Rightarrow 4^{1/2}\left(1-\frac{x}{4}\right)^{1/2}$

$\Rightarrow 2\left(1-\frac{x}{4}\right)^{1/2}$

$\Rightarrow 2\left\{1 + \left(\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(-\frac{x}{4}\right)^2}{2!} + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(-\frac{x}{4}\right)^3}{3!}\right\}$

4.2 Expanding $(a+bx)^n$

1 bi] Cont.

$= 2\left(1 - \frac{1}{8}x - \frac{1}{128}x^2 - \frac{1}{1024}x^3\right)$

$= 2 - \frac{1}{4}x - \frac{1}{64}x^2 - \frac{1}{512}x^3$

ii] $4 - x < 0$

$|x| < 4$

ci] $(8 + 2x)^{2/3} \Rightarrow 8^{2/3}\left(1 + \frac{1}{4}x\right)^{2/3}$

$= 4\left(1 + \frac{1}{4}x\right)^{2/3}$

$\Rightarrow 4\left(1 + \left(\frac{2}{3}\right)\left(\frac{1}{4}x\right) + \frac{\left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)}{2!}\left(\frac{1}{4}x\right)^2 + \frac{\left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)\left(\frac{2}{3}-2\right)}{3!}\left(\frac{1}{4}x\right)^3\right)$

$\Rightarrow 4\left(1 + \frac{1}{6}x - \frac{1}{144}x^2 + \frac{1}{1296}x^3\right)$

$\Rightarrow 4 + \frac{2}{3}x - \frac{1}{36}x^2 + \frac{1}{324}x^3$

ii] $8 + 2x < 0$

$2x < -8$

$|x| < \frac{8}{2}$

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4.2 Expanding $(a+bx)^n$

2a i] $\frac{3}{5-2x} = 3(5-2x)^{-1} \Rightarrow 3\times 5^{-1}\left(1-\frac{2}{5}x\right)^{-1}$

$\Rightarrow \frac{3}{5}\left(1-\frac{2}{5}x\right)^{-1}$

$\Rightarrow \frac{3}{5}\left(1+(-1)\left(-\frac{2}{5}x\right)+\frac{(-1)(-1-1)}{2!}\left(-\frac{2}{5}x\right)^2+\frac{(-1)(-1-1)+(-1-2)}{3!}\left(-\frac{2}{5}x\right)^3\right)$

$\Rightarrow \frac{3}{5}\left(1+\frac{2}{5}x+\frac{4}{25}x^2+\frac{8}{125}x^3\right)$

$= \frac{3}{5}+\frac{6}{25}x+\frac{12}{125}x^2+\frac{24}{625}x^3$

ii] $5-2x<0$

$|x|<\frac{5}{2}$

bi] $\frac{1-x}{(2-x)^2} = (1-x)(2-x)^{-2} \Rightarrow (1-x)2^{-2}\left(1-\frac{x}{2}\right)^{-2}$

$\Rightarrow \frac{1}{4}(1-x)\left(1-\frac{x}{2}\right)^{-2}$

$\Rightarrow \frac{1}{4}(1-x)\left(1+(-2)\left(-\frac{x}{2}\right)+\frac{(-2)(-2-1)}{2!}\left(-\frac{x}{2}\right)^2+\frac{(-2)(-2-1)(-2-2)}{3!}\left(-\frac{x}{2}\right)^3\right)$

$= \left(\frac{1}{4}-\frac{1}{4}x\right)\left(1+x+\frac{3}{4}x^2+\frac{1}{2}x^3\right)$

$= \frac{1}{4}+\frac{1}{4}x+\frac{3}{16}x^2+\frac{1}{8}x^3-\frac{1}{4}x-\frac{1}{4}x^2-\frac{3}{16}x^3$

$= \frac{1}{4}-\frac{1}{16}x^2-\frac{1}{16}x^3$

ii] $2-x<0$

$|x|<2$

4.2 Expanding $(a+bx)^n$

2ci] $\sqrt{\frac{4+x}{1-x}} \Rightarrow (4+x)^{1/2}(1-x)^{-1/2} \Rightarrow 4^{1/2}\left(1+\frac{x}{4}\right)^{1/2}(1-x)^{-1/2}$

$\Rightarrow 2\left(1+\frac{x}{4}\right)^{1/2}(1-x)^{-1/2}$

$\Rightarrow \left(1+\frac{x}{4}\right)^{1/2} = \left(1+\left(\frac{1}{2}\right)\left(\frac{x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}\left(\frac{x}{4}\right)^2+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\frac{x}{4}\right)^3\right)$

$\Rightarrow 1+\frac{x}{8}-\frac{1}{128}x^2+\frac{1}{1024}x^3$

$\Rightarrow (1-x)^{-1/2} = 1+\left(\frac{-1}{2}\right)(-x)+\frac{\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)}{2!}(-x)^2+\frac{\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)\left(\frac{-1}{2}-2\right)}{3!}(-x)^3$

$= 1+\frac{x}{2}+\frac{3}{8}x^2+\frac{3}{16}x^3$

$\Rightarrow 2\left(1+\frac{x}{8}-\frac{1}{128}x^2+\frac{1}{1024}x^3\right)\left(1+\frac{x}{2}+\frac{3}{8}x^2+\frac{3}{16}x^3\right)$

$\Rightarrow 2\left(1+\frac{x}{2}+\frac{3}{8}x^2+\frac{3x^3}{16}+\frac{x}{8}+\frac{x^2}{16}+\frac{3x^3}{64}-\frac{1}{128}x^2-\frac{1}{256}x^3+\frac{1}{1024}x^3\right)$

$\Rightarrow 2\left(1+\frac{5}{8}x+\frac{55}{128}x^2+\frac{365}{1024}x^3\right)$

$\Rightarrow 2+\frac{5}{4}x+\frac{55}{64}x^2+\frac{365}{512}x^3$

ii] $1-x<0$

$|x|<1$

BF MATHS

4.2 Expanding $(a+bx)^n$

3a] $f(x) = (5-3x)^{-3} \Rightarrow 5^{-3}\left(1-\frac{3}{5}x\right)^{-3}$

$\Rightarrow \frac{1}{125}\left(1-\frac{3}{5}x\right)^{-3}$

$\Rightarrow \frac{1}{125}\left(1+(-3)\left(\frac{-3}{5}x\right)+\frac{(-3)(-3-1)}{2!}\left(\frac{-3}{5}x\right)^2+\frac{(-3)(-3-1)(-3-2)}{3!}\left(\frac{-3}{5}x\right)^3\right)$

$\Rightarrow \frac{1}{125}\left(1+\frac{9}{5}x+\frac{54}{25}x^2+\frac{54}{25}x^3\right)$

$\Rightarrow \frac{1}{125}+\frac{9}{625}x+\frac{54}{3125}x^2+\frac{54}{3125}x^3$

b] $5-3x<0$

$|x|<\frac{5}{3}$

4a] $\frac{1}{\sqrt{6+2x}} \Rightarrow (6+2x)^{-1/2} \Rightarrow 6^{-1/2}\left(1+\frac{1}{3}x\right)^{-1/2}$

$\Rightarrow \frac{1}{\sqrt{6}}\left(1+\frac{1}{3}x\right)^{-1/2}$

$\Rightarrow \frac{1}{\sqrt{6}}\left(1+\left(\frac{-1}{2}\right)\left(\frac{1}{3}x\right)+\frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}-1\right)\left(\frac{1}{3}x\right)^2}{2!}\right)$

$\Rightarrow \frac{1}{\sqrt{6}}\left(1-\frac{1}{6}x+\frac{1}{24}x^2\right)$

$\Rightarrow \frac{1}{\sqrt{6}}-\frac{1}{6\sqrt{6}}x+\frac{1}{24\sqrt{6}}x^2$

b] $\frac{3x-1}{\sqrt{6+2x}} \Rightarrow (3x-1)(6+2x)^{-1/2}$

BE MATHS

4.2 Expanding $(a+bx)^n$

4b] Cont.

$$(3x-1)\left(\frac{1}{\sqrt{6}} - \frac{1}{6\sqrt{6}}x + \frac{1}{24\sqrt{6}}x^2\right)$$

$$\Rightarrow \frac{3x}{\sqrt{6}} - \frac{1}{2\sqrt{6}}x^2 - \frac{1}{\sqrt{6}} + \frac{1}{6\sqrt{6}}x - \frac{1}{24\sqrt{6}}x^2$$

$$\Rightarrow -\frac{1}{\sqrt{6}} + \frac{19\sqrt{6}}{36}x - \frac{13}{24\sqrt{6}}x^2$$

5a] $\dfrac{1}{\sqrt{a+bx}} \Rightarrow (a+bx)^{-1/2} \Rightarrow a^{-1/2}\left(1+\frac{bx}{a}\right)^{-1/2}$

$$\Rightarrow a^{-1/2}\left(1 + \left(\frac{-1}{2}\right)\left(\frac{b}{a}x\right) + \frac{\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)\left(\frac{b}{a}x\right)^2}{2!}\right)$$

$$\Rightarrow \frac{1}{\sqrt{a}}\left(1 - \frac{b}{2a}x + \frac{3}{8}\left(\frac{b^2}{a^2}\right)x^2\right)$$

$$\Rightarrow \frac{1}{\sqrt{a}} - \frac{b}{2a\sqrt{a}}x + \frac{3}{8\sqrt{a}}\left(\frac{b^2}{a^2}\right)x^2$$

BF MATHS

Equating Constant

$$\Rightarrow \frac{1}{\sqrt{a}} = \frac{1}{3}$$

$$\Rightarrow \sqrt{a} = 3$$

$$\Rightarrow a = 3^2$$

$$\Rightarrow \underline{a = 9}$$

Equating x

$$\Rightarrow -\frac{b}{2a\sqrt{a}} = -\frac{2}{27}$$

$$\Rightarrow \frac{b}{2\times 9\times\sqrt{9}} = \frac{2}{27}$$

4.2 Expanding $(a+bx)^n$

5a] Cont.

$$\frac{b}{54} = \frac{2}{27}$$

$$27b = 108$$

$$b = 4$$

$\therefore a = 9$ and $b = 4$

b] $\frac{1}{\sqrt{9+4x}}$

$\Rightarrow 9 + 4x < 0$

$|x| < \frac{9}{4}$

c] $(9+4x)^{-1/2} \Rightarrow 9^{-1/2}\left(1+\frac{4}{9}x\right)^{-1/2}$

$\Rightarrow \frac{1}{3}\left(1+\frac{4}{9}x\right)^{-1/2}$

$\Rightarrow \frac{1}{3}\left(\frac{\left(\frac{-1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}\left(\frac{4}{9}x\right)^3\right)$

$\Rightarrow \frac{1}{3}\left(-\frac{20}{729}x^3\right)$

$\Rightarrow -\frac{20}{2187}x^3$

$\therefore$ The constant is $-\frac{20}{2187}$

BF MATHS

4.2 Expanding $(a+bx)^n$

6a] $(8-5x)^{1/2} \Rightarrow 8^{1/3}\left(1-\frac{5}{8}x\right)^{1/2}$

$\Rightarrow 2\left(1-\frac{5}{8}x\right)^{1/3}$

$\Rightarrow 2\left(1+\left(\frac{1}{3}\right)\left(\frac{-5}{8}x\right)+\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)\left(\frac{-5}{8}x\right)^2}{2!}\right)$

$\Rightarrow 2-\frac{5}{12}x-\frac{25}{288}x^2$

6b] $(8-5x)^{1/3} = \sqrt[3]{7}$

$8-5x = 7$

$8-7 = 5x$

$1 = 5x$

$x = 0.2$

$\sqrt[3]{7} = 2-\frac{5}{12}(0.2)-\frac{25}{288}(0.2)^2$

$\sqrt[3]{7} = 1.913$

7a] $g(x) = \frac{4}{2-4x} - \frac{2}{3-5x}$

$\Rightarrow 4(2-4x)^{-1} - 2(3-5x)^{-1}$

$= 4\times 2^{-1}(1-2x)^{-1} - 2\times 3^{-1}\left(1-\frac{5}{3}x\right)^{-1}$

$= 2(1-2x)^{-1} - \frac{2}{3}\left(1-\frac{5}{3}x\right)^{-1}$

$= 2\left(1+(-1)(-2x)+\frac{(-1)(-1-1)(-2x)^2}{2!}\right)$

$= 2(1+2x+4x^2)$

4.2 Expanding $(a+bx)^n$

DATE / /
PAGE

7a] Cont.

$= 2 + 4x + 8x^2$

$= -\frac{2}{3}\left(1 + (-1)\left(\frac{-5x}{3}\right) + \frac{(-1)(-1-1)}{2!}\left(\frac{-5x}{3}\right)^2\right)$

$= -\frac{2}{3}\left(1 + \frac{5x}{3} + \frac{25x^2}{9}\right)$

$= -\frac{2}{3} - \frac{10}{9}x - \frac{50x^2}{27}$

$= 2 + 4x + 8x^2 - \frac{2}{3} - \frac{10}{9}x - \frac{50}{27}x^2$

$= \frac{4}{3} + \frac{26}{9}x + \frac{166}{27}x^2$

b] $2 - 4x < 0$

$|x| < \frac{1}{2}$

c] $g(x) = \frac{4}{2-4(0.01)} - \frac{2}{3-5(0.01)}$

$= 1.3628502$

d] when $x = 0.01$

$= \frac{4}{3} + \frac{26}{9}(0.01) + \frac{166}{27}(0.01)^2$

$= 1.362837037$

Percentage error formula $\Rightarrow \frac{\text{estimate} - \text{correct}}{\text{correct}} \times 100$

BF MATHS

4.2 Expanding $(a+bx)^n$

8a) Cont.

$$\Rightarrow \left(1 + \frac{1}{3}x - \frac{1}{9}x^2\right)\left(\frac{1}{2} + \frac{1}{16}x + \frac{1}{64}x^2\right)$$

$$\Rightarrow \frac{1}{2} + \frac{1}{16}x + \frac{1}{64}x^2 + \frac{1}{6}x + \frac{1}{48}x^2 - \frac{1}{18}x^2$$

$$\Rightarrow \frac{1}{2} + \frac{11}{48}x - \frac{11}{576}x^2$$

b) $1 + x < 0$

$|x| < 1$

c) $x = 0.1 \Rightarrow \frac{1}{2} + \frac{11(0.1)}{48} - \frac{11(0.1)^2}{576}$

$$\Rightarrow 0.522725694 = 3\sqrt{\frac{1+0.1}{8-3(0.1)}}$$

$$\Rightarrow 0.522725694 = 3\sqrt{\frac{1}{7}}$$

$$\Rightarrow \frac{1}{0.52272\ldots} = 3\sqrt{7}$$

$\Rightarrow$ 1.91305 (5dp)

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