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5.3 - Areas of sectors and segments

① a) $r = 5\text{cm}$ $\theta = 0.45$ rad $A = ?$

$\Rightarrow A = \frac{1}{2}r^2\theta \Rightarrow \frac{1}{2}(5)^2(0.45) = 5.625\ \text{cm}^2 = 5.63\ \text{cm}^2$ (3sf)

b) $r = 7.3\text{cm}$ $\theta = 1.2$ rad $A = ?$

$\Rightarrow A = \frac{1}{2}r^2\theta = \frac{1}{2}(7.3)^2(1.2) = 32.0\ \text{cm}^2$ (3sf)

c) $r = r$ $A = \frac{9\pi}{8}\ \text{cm}^2$ $\theta = \frac{\pi}{4}$

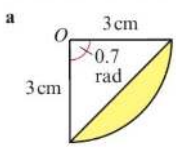
$\Rightarrow A = \frac{1}{2}r^2\theta \Rightarrow \frac{9\pi}{8} = \frac{1}{2}(r^2)\left(\frac{\pi}{4}\right) \Rightarrow r^2 = 9$ $r = 3$

d) $\theta = \theta$ $r = 7.2$ $A = 33.696\ \text{cm}^2$

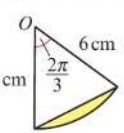
$\Rightarrow A = \frac{1}{2}r^2\theta \Rightarrow 33.696 = \frac{1}{2}(7.2)^2\theta \Rightarrow \theta = 1.3$ rad

② 2 Find the area of the shaded segment in each of these sectors.

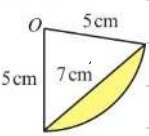
a



b



c



a) $A = \frac{1}{2}r^2(\theta - \sin\theta) = \frac{1}{2}(3)^2(0.7 - \sin(0.7)) = 0.251\ \text{cm}^2$ (3sf)

b) $A = \frac{1}{2}r^2(\theta - \sin\theta) = \frac{1}{2}(6)^2\left(\frac{2\pi}{3} - \sin\left(\frac{2\pi}{3}\right)\right) = 22.1\ \text{cm}^2$ (3sf)

c) $\angle O = \cos^{-1}\left(\frac{5^2+5^2-7^2}{2(5)(5)}\right) = 1.55\ldots \Rightarrow \frac{1}{2}(5)^2(1.55\ldots - \sin(1.55\ldots))$

$= 6.89\ \text{cm}^2$ (3sf)

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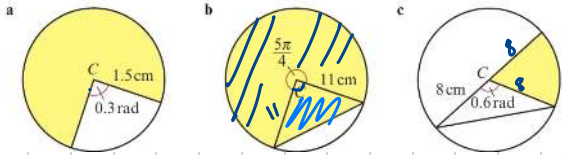
3 Find the shaded area in each of these circles with centre C.



(3)

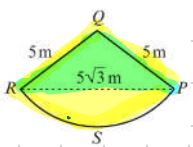
a) $2\pi - 0.3\text{rad} = 5.98 \Rightarrow A = \frac{1}{2}r^2\theta = \frac{1}{2}(1.5)^2(5.98\ldots) = 6.73\,cm^2$ (3sf)

b) A of major sector $= \frac{1}{2}(11)^2\left(\frac{5\pi}{4}\right) = \frac{605\pi}{8}$ Area of $\Delta = \frac{1}{2}(11)^2(\sin(2\pi - \frac{5\pi}{4})$

$\Rightarrow \frac{605\pi}{8} + 42.7799\ldots = 280\,cm^2$ (3sf) $\Rightarrow 42.7799\ldots$

c) Area of major sector − Area of $\Delta \Rightarrow \left[\frac{1}{2}(8)^2(2\pi - 0.6)\right] - \frac{\pi}{2}(8)^2 = 81.3\,cm^2$ (3sf)

(4)



$r = 5$ $PR = 5\sqrt{3}$

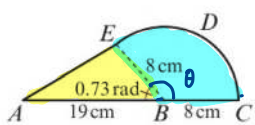
a) $\angle PQR \Rightarrow \cos^{-1}\left(\frac{5^2+5^2-(5\sqrt{3})^2}{2(5)(5)}\right) = \frac{2\pi}{3}$ rad.

b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(5)^2\left(\frac{2\pi}{3}\right) = \frac{25\pi}{3}\,m^2$

c) Area of segment PRS = Area of Δ − Area of $\Delta = \frac{25\pi}{3} - \frac{1}{2}(5^2)(\sin(\frac{2\pi}{3}))$

$= 15.4\,m^2$ (3sf)

(5)



$r = 8$; $AB = 19\,cm$ $BC = 8\,cm$ $\angle ABE = 0.73\,rad = x$

a) Area of shape = Area of Δ + Area of sector

$\Rightarrow \frac{1}{2}ab\sin x + \frac{1}{2}r^2\theta \Rightarrow \frac{1}{2}(19)(8)(\sin(0.73)) + \frac{1}{2}(8)^2(\pi - 0.73)$

$= 128\,cm^2$ (3sf)

b) perimeter of shape = $19 + 8 + AE + EDC \cap (l)$

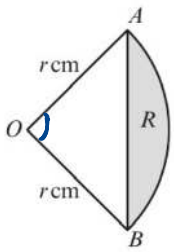
$\Rightarrow l = r\theta = 8(\pi - 0.73) = 19.293\ldots$

$\Rightarrow AE = \sqrt{(8)^2 + (19)^2 - 2(8)(19)\cos(0.73)} = 14.0878\ldots$

$\Rightarrow 19 + 8 + 19.293\ldots + 14.0878\ldots$

$= 60.4\,cm$ (3sf)

(6)



$r = r$; Area of $\Delta = 37.8\ cm^2$, $\angle AOB = 1.2$ rad.

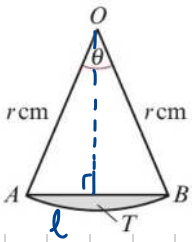
a) $A = \frac{1}{2}r^2\theta \Rightarrow 37.8 = \frac{1}{2}(r^2)(1.2) \Rightarrow r^2 = 63$

$\Rightarrow r = 3\sqrt{7}$

b) R = Area of Δ − Area of $\Delta = 37.8 - \frac{1}{2}(3\sqrt{7})^2 \sin(1.2)$

$\Rightarrow$ area of $R = 8.4\ cm^2$ (1dp)

(7)



$\angle AOB = \theta$; Area of $\Delta AOB = 12\ cm^2$

a) perimeter of the sector = 6 (length of arc)

$\Rightarrow r + r + l = 6(l)$ $\quad$ $l = r\theta$

$\Rightarrow r + r + r\theta = 6r\theta$

$\Rightarrow r(1+1+\theta) = 6r\theta \Rightarrow 2 + \theta = 6\theta$

$\Rightarrow 2 = 6\theta - \theta \Rightarrow \theta(6-1) = 2 \Rightarrow \theta = \frac{2}{5}$

$\Rightarrow A = \frac{1}{2}r^2\theta \Rightarrow 12 = \frac{1}{2}r^2\left(\frac{2}{5}\right) \Rightarrow 60 = r^2 \Rightarrow r = 2\sqrt{15}$ cm

b) Area of T = Area of sector AOB − Area of Δ AOB.

$\Rightarrow 12 - \frac{1}{2}(2\sqrt{15})^2\left(\sin\left(\frac{2}{5}\right)\right) = 0.317\ cm^2$ (3sf)