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3.3 Geometric sequences

(1) a) Given: 2, 6, 18, 54, ...

i) $r = 3$ ii) $a = 2$ $r = 3$ $u_n = 2(3)^{n-1}$

b) Given: 60, 30, 15, 7.5 ....

i) $r = \frac{1}{2}$ ii) $a = 60$ $r = \frac{1}{2}$ $u_n = 60\left(\frac{1}{2}\right)^{n-1}$

c) Given: 10, −20, 40, −80

i) $r = -2$ ii) $a = 10$ $r = -2$ $u_n = 10(-2)^{n-1}$

d) Given: 40, 60, 90, 135 ....

i) $r = 3/2$ ii) $a = 40$ $r = 3/2$ $u_n = 40(3/2)^{n-1}$

(2) a) i.e. 10, −20, 40, −80

b) Let $p = ar^k$, $q = ar^{k+1}$, $r = ar^{k+2}$

$pr = ar^k \times ar^{k+2} = a^2r^{2k+2} = (ar^{k+1})^2 = q^2 \Rightarrow pr = q^2$

(3) Given: $a = 5$ $ar = x$ $ar^2 = x + 10$

a) $\frac{ar^2}{ar} = \frac{ar}{a} \Rightarrow \frac{x+10}{x} = \frac{x}{5} \Rightarrow 5x + 50 = x^2 \Rightarrow x^2 - 5x - 50 = 0$

$x = 10$ or ~~$x = -5$~~ $a = 5$ $r = 2$

b) $u_{10} = ar^{n-1} = 5(2)^9 = 2560$

(4) Given: 4, 12, 36, 108, ... $u_n > 1000\,000$ $a = 4$ $ar = 12$ $r = 3$

a) $4(3)^{n-1} > 1000\,000$ $3^{n-1} > 250000$ $n - 1 > 11.3$

$n > 12.3$ $n = 13$ $\Rightarrow u_{13} = 4(3)^{12} = 2125764$

(5) Given: $ar^3 = 16$, $ar^6 = 250$

a) $\frac{ar^6}{ar^3} = \frac{250}{16} \Rightarrow r^3 = \frac{125}{8}$ $r = \frac{5}{2}$

b) $a\left(\frac{5}{2}\right)^3 = 16$ $a = \frac{128}{125}$

(6) Given: $a = k + 4$ $ar = 3k$ $ar^2 = 2k^2$

$\frac{ar}{a} = \frac{ar^2}{ar} \Rightarrow \frac{3k}{k+4} = \frac{2k^2}{3k} \Rightarrow 9k^2 = 2k^3 + 8k^2 \Rightarrow 2k^3 - k^2 \Rightarrow k^2(2k - 1)$

~~$k = 0$~~ or $k = \frac{1}{2}$

(7) Given: $u_3 = ar^2 = 108$ $a_6 = ar^5 = 32$

a) $\frac{ar^5}{ar^2} = \frac{32}{108} \Rightarrow r^3 = \frac{8}{27} \Rightarrow r = \frac{2}{3}$ $a = \frac{108 \times 9}{4} = 243$

$u_{20} = (243)\left(\frac{2}{3}\right)^{19} = 0.110$ (3sf)

b) $u_n < 1 \Rightarrow (243)\left(\frac{2}{3}\right)^{n-1} < 1 \Rightarrow \left(\frac{2}{3}\right)^{n-1} < \frac{1}{243} \Rightarrow n-1 < 13.6$

$n < 14.6$ $\boxed{n = 15}$ $u_{15} = ar^{14} = (243)\left(\frac{2}{3}\right)^{14} = 0.832$ (3sf)

(8) Given: $a = 150$ $r = r$ $ar^4 = 50$

a) $150r^4 = 50 \Rightarrow r^4 = \frac{1}{3}$ $4\ln r = \ln\frac{1}{3} \Rightarrow 4\ln r = \ln 1 - \ln 3$ (0 ↑)

$\Rightarrow 4\ln r = -\ln 3 \Rightarrow 4\ln r + \ln 3 = 0$

b) $r = \sqrt[4]{\frac{1}{3}}$ $r = 0.760$ (3sf)

(9) Given: $a = 5k$ $ar = 4k+3$ $ar^2 = 6k+7$

a) $\frac{ar^2}{ar} = \frac{ar}{a} \Rightarrow \frac{6k+7}{4k+3} = \frac{4k+3}{5k} \Rightarrow 5k(6k+7) = (4k+3)^2$

$\Rightarrow 30k^2 + 35k = 16k^2 + 9 + 24k$

$\Rightarrow 14k^2 + 11k - 9 = 0 \Rightarrow k = \frac{1}{2}, k > 0$

b) $\boxed{a = \frac{5}{2}}$ $ar = 11 \Rightarrow \frac{5}{2}r = 5$ $\boxed{r = 2}$

$ar^{n-1} > 10000 \Rightarrow \frac{5}{2}(2)^{n-1} > 10000 \Rightarrow (2)^{n-1} > 4000$

$n-1 > 11.96$ $n > 12.96$ $\boxed{n = 13}$, $u_{13} = ar^{12} = \frac{5}{2}(2)^{12} = 10240$