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3.2 Arithmetic series

$$S_n = \frac{n}{2}\left(2a+(n-1)d\right) \quad \text{or} \quad \frac{n}{2}(a+L)$$

1 a) Given: $a=5$ $d=4$ $n=20$

$$S_{20} = \frac{20}{2}\left(2(5)+19(4)\right) = 860$$

b) Given: $a=3$ $d=5$ $n=15$

$$S_{15} = \frac{15}{2}\left(2(3)+14(5)\right) = 570$$

c) Given: $a=12$ $d=-2$ $n=40$

$$S_{40} = \frac{40}{2}\left(2(12)+39(-2)\right) = -1080$$

d) Given: $a=8$ $d=-0.5$ $n=25$

$$S_{25} = \frac{25}{2}\left(2(8)+24(-0.5)\right) = 50$$

2 a) Given: $a=6$ $d=3$ $L=51$

$$u_n = a+(n-1)d \Rightarrow 51 = 6+(n-1)3$$

$$48 = 3n \quad \boxed{n=16}$$

$$S_n = \frac{16}{2}\left(2(6)+15(3)\right) = 456$$

b) Given: $a = 52$ $d = -5$ $L = -103$ $n = ?$ $S_n = ?$

$-103 = 52 + (n-1)-5 \Rightarrow -103 = 57 - 5n$

$\cancel{-}160 = \cancel{-}5n$ $\boxed{n = 32}$

$S_n = \frac{32}{2}\left(2(52) + 31(-5)\right) = -816$

c) Given: $a = -2$ $d = 3$ $L = 175$

$175 = -2 + (n-1)3 \Rightarrow 175 = 3n - 5$

$180 = 3n$ $\boxed{n = 60}$

$S_n = 30\left(2(-2) + 59(3)\right) = 5190$

d) Given: $a = 120$ $d = -8$ $L = 0$

$0 = 120 + (n-1)-8$ $0 = 128 - 8n$

$128 = 8n$ $\boxed{n = 16}$

$S_n = 8\left(2(120) + (15)(-8)\right) = 960$

3 Given: $n = 60$ $S_{60} = 1830$

$a = 1$ $L = 60$ $S_n = \frac{60}{2}(1 + 60) = 30(61) = 1830$

$\therefore$ Proved.

④ Given: $a=5$ $d=3$ $n=?$ $S_n > 1000$

$$S_n = \frac{n}{2}\left(2(5)+(n-1)3\right) > 1000$$

$$= \frac{n}{2}\left(10-3+3n\right) > 1000$$

$$= \frac{n}{2}\left(7+3n\right) > 1000$$

$$= 3n^2+7n-2000 > 0$$

$$= n > 24.67 \Rightarrow \boxed{n=25}$$

⑤ Given: $4r-5$

a) $a_1 = 4(1)-5 = -1$ $a_2 = 4(2)-5 = 3$ $a_3 = 4(3)-5 = 7$

b) $d = 4$

c) $S_n = n(2n-3)$ $d=4$, $a=-1$

$$S_n = \frac{n}{2}\left(2(-1)+(n-1)4\right) = \frac{n}{2}\left(-2-4+4n\right)$$

$$\Rightarrow \frac{n}{2}\left(-6+4n\right) \Rightarrow n(2n-3)$$

⑥ Given :- $S_n = \frac{n(n+1)}{2}$

$1+2+3+\dots+(n-2)+(n-1)+n$

$S_n = \frac{n}{2}(a+L)$ $a=1$, $L=n$ $S_n = \frac{n(n+1)}{2}$

$\therefore$ Proved

b) Given: sum of even integers (2 to 200)

$a = 2 \quad L = 200 \qquad 2, 4, 6, 8 .. 200 \quad d = 2$

$u_n = 2 + (n-1)2 \qquad 200 = 2 - 2 + 2n \quad \boxed{n = 100}$

$S_n = \frac{100}{2}(2+200) = 50(202) = 10100$

7 Given: $u_1, u_2, \ldots u_n$

a) $u_4 = 103 \quad u_{12} = 79 \qquad u_{25} = ?$

$\cancel{a} + 3d = 103 \quad — ①$

$\cancel{a} + 11d = 79 \quad — ②$

$-8d = 24$

$\boxed{d = -3} \Rightarrow a + 3(-3) = 103$

$\boxed{a = 112}$

$a_{25} = a + 24d = 112 + 24(-3) = 40$

b) Given: $S_n = 1925 \quad a = 112 \quad d = -3 \quad n = ?$

$\Rightarrow 1925 = \frac{n}{2}\left(2(112) + (n-1)-3\right)$

$\Rightarrow 3850 = n(227 - 3n)$

$\Rightarrow +3n^2 - 227n + 3850 = 0 \qquad \boxed{n = 50} \quad \text{or} \quad \cancel{n = \frac{77}{3}}$

8 Given: $n = 12 \quad S_n = 366$

a) $366 = \frac{12}{2}(2a + 11d) \Rightarrow 366 = 6(2a + 11d) \Rightarrow 366 = 12a + 66d$

Given: $a_8 = 38 \qquad 12a + 66d = 366 \quad — ①$

b) $a + 7d = 38 \quad — ②$

$(a+7d=38)\times 12$

c) $\cancel{12a}+66d=366$ $\quad 12a=366-66(5)$

$\cancel{12a}+84d=456$ $\quad \boxed{a=3}$

$-18d=-90$

$\boxed{d=5}$

(9) Given: $a_2+a_6=14$

a) $\underbrace{a+d}_{a_2}+\underbrace{a+5d}_{a_6}=14$ $\quad \Rightarrow (2a+6d=14)\div 2$

$\Rightarrow a+3d=7 \quad -(1)$

b) Given: $S_{20}=-120 \quad n=20$

$-120=10(2a+19d) \Rightarrow -120=20a+190d \quad -(2)$

(1) $\times 20$ (2) $\times 1$

$\boxed{d=-2}$

$\Rightarrow \cancel{20a}+60d=140$

$\cancel{20a}+190d=-120$

$-13\cancel{0}d=+26\cancel{0}$

$a+3(-2)=7$

$a=7+6 \quad \boxed{a=13}$

(10) Given: $a_{15}=11 \quad a_{20}=\frac{37}{2}$

$\cancel{a}+14d=11$

$\cancel{a}+19d=\frac{37}{2}$

$5d=\frac{15}{2}$

$\boxed{d=\frac{3}{2}}$

$a+\cancel{14}^{7}\left(\frac{3}{\cancel{2}}\right)=11$

$a=11-21 \quad \boxed{a=-10}$

$a_2=a+d=-10+\frac{3}{2}=\frac{-17}{2}$

b) Given: $a = -10$ $S_n = 2055$ $d = \frac{3}{2}$

$$2055 = \frac{n}{2}\left(-20 + (n-1)\frac{3}{2}\right)$$

$$4110 = n\left(-\frac{43}{2} + \frac{3}{2}n\right) \Rightarrow 4110 = -\frac{43}{2}n + \frac{3}{2}n^2$$

$$\Rightarrow 8220 = -43n + 3n^2$$

$$\Rightarrow 3n^2 - 43n - 8220 = 0 \qquad \boxed{n = 60} \qquad n > 0$$

(11) Given: $a_2 = 3k$ $a_3 = 4k+5$ $a_4 = 7k$ $S_n = kn^2$

$d_1 = 4k+5 - 3k = k+5$ $\qquad d_1 = d_2$

$d_2 = 7k - 4k - 5 = 3k - 5$ $\qquad k+5 = 3k-5$

$10 = 2k$

$\boxed{k = 5}$

$a_2 = 15,\ a_3 = 25,\ a_4 = 35$

$d = 10$ $\qquad \boxed{a = 5}$

$$S_n = \frac{n}{2}\left(10 + 10(n-1)\right) = \frac{\cancel{10}^{5}n}{\cancel{2}}\left(\cancel{1} + (n - \cancel{1})\right) = 5n^2$$

(12) Given: $a_3 = 3k$ $a_4 = 4k+3$ $a_5 = 6k-9$

$d_1 = 4k+3-3k = k+3$ $\qquad d_1 = d_2$

$d_2 = 6k-9-4k-3 = 2k-12$

$k+3 = 2k-12$

$\boxed{15 = k}$

$a_3 = 45,\ a_4 = 63,\ a_5 = 81$

$\boxed{a = 9}$ $\qquad \boxed{d = 18}$

$$S_n = \frac{n}{2}\left(18 + (n-1)18\right) = \frac{n}{2}\left(\cancel{18} + 18n - \cancel{18}\right)$$

$$S_n = \frac{n}{\cancel{2}}\left(\overset{9}{\cancel{18}}n\right) = 9n^2 = (3n)^2$$

So the sum of first n terms of the sequence is a square number