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14.8 Logarithms and non-linear data

1.

a) $V = 5x^3$

take logs of both sides

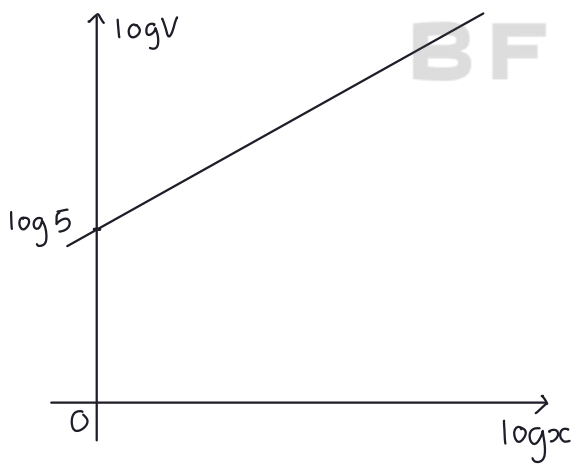
$\log V = \log 5x^3$

$\log V = \log 5 + \log x^3$

$\log V = \log 5 + 3\log x$

$\therefore \log_{10} V = \log_{10} 5 + 3\log_{10} x$

b) positive straight line with y-intercept $= \log 5$



c) $\log V = \log 5 + 3\log x$

$y \quad c \quad m \quad x$

$\therefore$ gradient $= 3$

2.

a) $H = 3\left(\frac{3}{2}\right)^t$

take logs of both sides

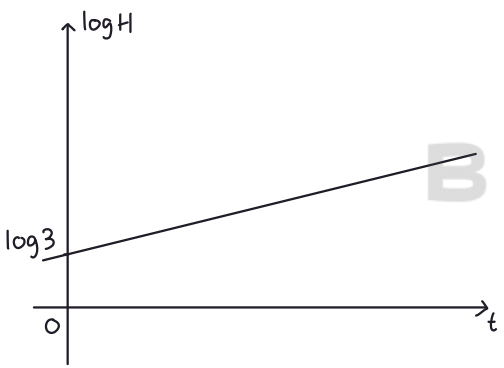
$\log H = \log 3\left(\frac{3}{2}\right)^t$

$\log H = \log 3 + \log\left(\frac{3}{2}\right)^t$

$\log H = \log 3 + t\log\frac{3}{2}$

$\therefore \log_{10} H = \log_{10} 3 + t\log_{10}\frac{3}{2}$

b) positive straight line with y-intercept $y = \log 3$



c) $\log H = \log 3 + t\log\frac{3}{2}$

$y \quad c \quad x \quad m$

$\therefore$ gradient $= \log\frac{3}{2}$

3.

a) $y = ab^x$

take logs of both sides

$\log y = \log ab^x$

$\log y = \log a + \log b^x$

$\log y = \log a + x \log b$

$\therefore \log_{10} y = \log_{10} a + x \log_{10} b$

b)

x	0.5	2	5	7
y	847.5	1878.5	9229.1	26672.0

$0.5 \rightarrow \log 847.5 = 2.93$

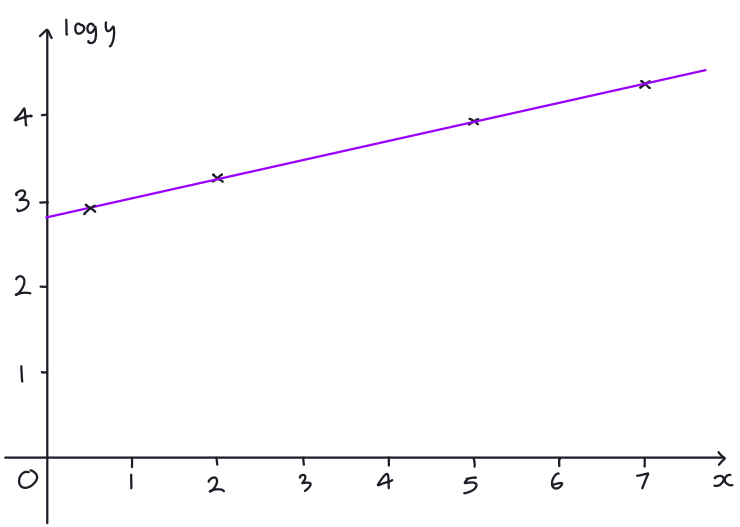
$2 \rightarrow \log 1878.5 = 3.27$

$5 \rightarrow \log 9229.1 = 3.97$

$7 \rightarrow \log 26672.0 = 4.43$

x	0.5	2	5	7
$\log y$	2.93	3.27	3.97	4.43

c)



d) $\log V = \log a + x \log b$

gradient $= \log_{10} b$

gradient from table $= \frac{4.43 - 2.93}{7 - 0.5} = \frac{3}{13}$

$\log_{10} b = \frac{3}{13}$

$b = 10^{\frac{3}{13}} = 1.70125...$

Reading intercept from graph,

$\log a = 2.81$

$a = 10^{2.81} = 645.654...$

$a = 650, \quad b = 1.7$

4.

a)

Year	2008	2010	2012	2014	2016	2018
Value, £V	25000	14823	8788	5211	3089	1832

$2008 \longrightarrow \log 25000 = 4.40$
$2010 \longrightarrow \log 14823 = 4.17$
$2012 \longrightarrow \log 8788 = 3.94$
$2014 \longrightarrow \log 5211 = 3.72$
$2016 \longrightarrow \log 3089 = 3.49$
$2018 \longrightarrow \log 1832 = 3.26$

Time in years since 2008, t	0	2	4	6	8	10
$\log_{10} V$	4.40	4.17	3.94	3.72	3.49	3.26

b) $V = ab^t$

take logs of both sides

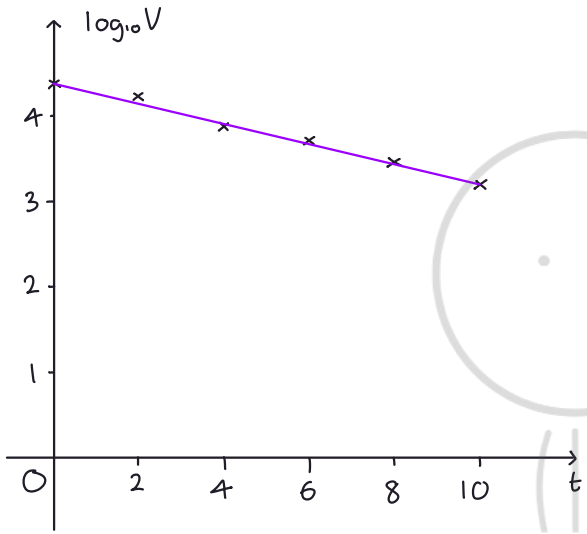
$\log V = \log ab^t$

$\log V = \log a + \log b^t$

$\log V = \log a + t \log b$

$\therefore \log_{10} V = \log_{10} a + t \log_{10} b$

c)



d) Draw line of best fit

gradient using table $= \frac{3.26 - 4.40}{10 - 0} = -0.114$

e) $\log V = \log a + t \log b$

gradient $= \log_{10} b$

$\log_{10} b = -0.114$

$b = 10^{-0.114} = 0.76913...$

Reading intercept from graph and table,

$\log a = 4.40$

$a = 10^{4.40} = 25118.86432$

$a = 25100, \quad b = 0.770$

5.

a)

Date	12/3/18	13/3/18	14/3/18	15/3/18	16/3/18
price, p pence	27.43	29.76	32.29	35.04	38.01

12/3/18 $\longrightarrow \log 27.43 = 1.44$
13/3/18 $\longrightarrow \log 29.76 = 1.47$
14/3/18 $\longrightarrow \log 32.29 = 1.51$
15/3/18 $\longrightarrow \log 35.04 = 1.54$
16/3/18 $\longrightarrow \log 38.01 = 1.58$

Time in days since 12/3/18, t	0	1	2	3	4
$\log_{10} P$	1.44	1.47	1.51	1.54	1.58

b) $P = ab^t$

take logs of both sides

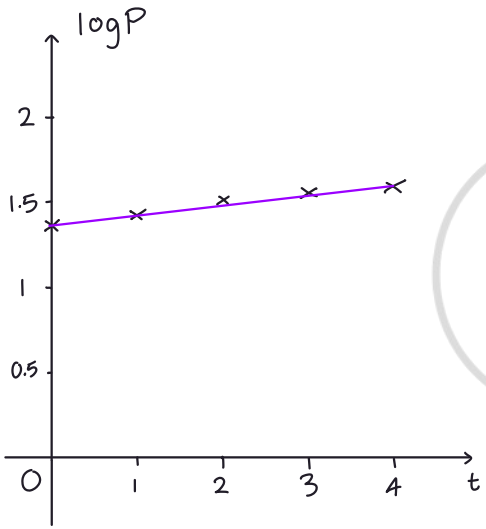
$\log P = \log ab^t$

$\log P = \log a + \log b^t$

$\log P = \log a + t \log b$

$\therefore \log_{10} P = \log_{10} a + t \log_{10} b$

c)



d) gradient from table $= \frac{1.58 - 1.44}{4 - 0} = \frac{7}{200}$

gradient $= \log_{10} b$

$\log_{10} b = \frac{7}{200}$

$b = 10^{\frac{7}{200}} = 1.083926...$

Reading intercept from graph and table,

$\log a = 1.44$

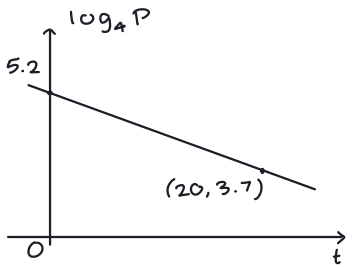
$a = 10^{1.44} = 27.54228...$

$a = 27.5, \quad b = 1.08$

* Answer may vary depending on y-intercept

6.

a)



$P = ab^t$

$\log P = \log ab^t$

$\log P = \log a + \log b^t$

$\log P = \log a + t \log b$

intercept $= 5.2$

gradient $= \dfrac{3.7 - 5.2}{20} = -0.075$

$\log_4 P = -0.075t + 5.2$

b) When $t = 0$

$\log_4 P = -0.075(0) + 5.2$

$\log_4 P = 5.2$

$P = 4^{5.2} = 1351.176101$

$P = 1350$

c) gradient $= \log_4 b$

$\log_4 b = -0.075$

$b = 4^{-0.075} = 0.90125...$

intercept $= \log_4 a$

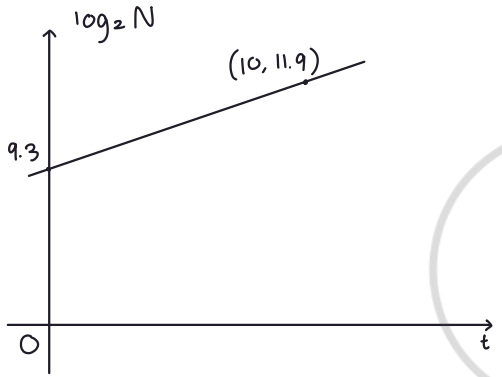
$\log_4 a = 5.2$

$a = 4^{5.2} = 1351.176101$

$a = 1350, \quad b = 0.901$

7.

a)



intercept = 9.3

gradient = $\frac{11.9 - 9.3}{10} = 0.26$

$\log_2 N = 0.26t + 9.3$

b) $N = ab^t$

$\log N = \log a + \log b^t$

$\log N = \log a + t\log b$

gradient = $\log_2 b$

$\log_2 b = 0.26$

$b = 2^{0.26} = 1.19747...$

intercept = $\log_2 a$

$\log_2 a = 9.3$

$a = 2^{9.3} = 630.3459...$

$a = 630.3$, $b = 1.2$

c) $a = 630.3$

so a is the initial number of bacteria in the petri dish