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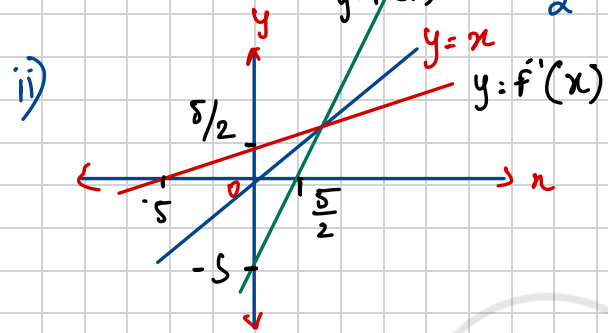
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d.4: Inverse functions

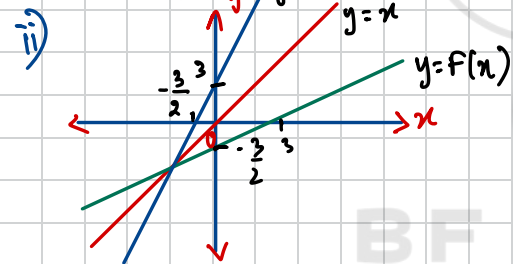
① a) $f: x \rightarrow 2x - 5, x \in \mathbb{R}$

i) $y = 2x - 5 \Rightarrow \frac{y+5}{2} = x \Rightarrow y = \frac{x+5}{2} \Rightarrow f^{-1}(x) = \frac{x+5}{2}$



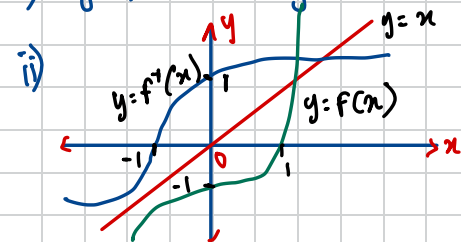
b) $f: x \rightarrow \frac{x-3}{2}, x \in \mathbb{R}$

i) $y = \frac{x-3}{2} \Rightarrow 2y+3 = x \Rightarrow f^{-1}(x) = 2x+3$



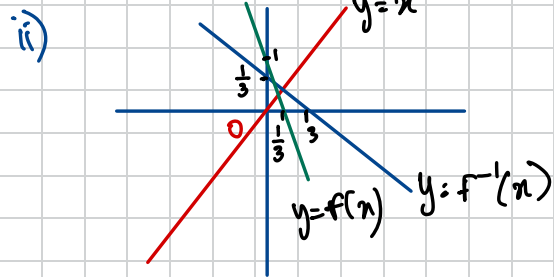
c) $f: x \mapsto x^3 - 1, x \in \mathbb{R}$

i) $y = x^3 - 1 \Rightarrow y+1 = x^3 \Rightarrow x^3 = 3\sqrt{y+1} = f^{-1}(x) = 3\sqrt{x+1}$



d) $f: x \mapsto 1-3x, x \in \mathbb{R}$

$y = 1-3x \Rightarrow 1-y = 3x \Rightarrow x = \frac{1-y}{3} \Rightarrow f^{-1}(x) = \frac{1-x}{3}$



(2) a) $g(x) = 2x - 3, x \in \mathbb{R}, x \geq 0$

i) $y = 2x - 3 \Rightarrow x = \frac{y+3}{2} \Rightarrow f^{-1}(x) = \frac{x+3}{2}$

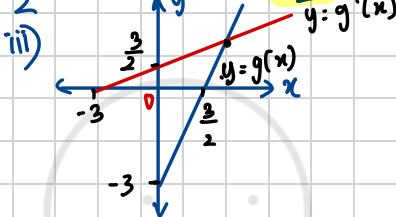
iv) $2x - 3 = \frac{x+3}{2}$

$\Rightarrow 4x - 6 = x + 3$

$\Rightarrow 3x = 9$

$x = 3$

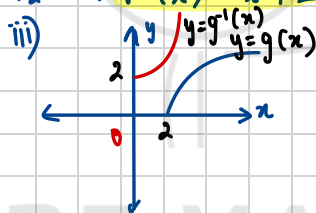
ii) Range: $g^{-1}(x) \geq 0$
Domain: $x \geq -3$



b) $g(x) = \sqrt{x-2}, x \in \mathbb{R}, x > 2$

i) $y = \sqrt{x-2} \Rightarrow x = y^2 + 2 \Rightarrow f^{-1}(x) = x^2 + 2$

ii) Range: $g^{-1}(x) > 2$
Domain: $x > 0$

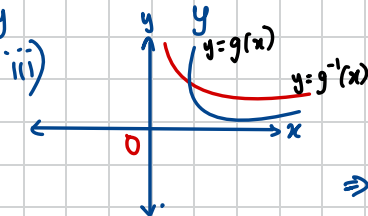


iv) $g(x) = g^{-1}(x)$ has no solutions, as the graph of $g(x)$ and $g^{-1}(x)$ does not intersect.

c) $g(x) = \frac{4}{x-3}, x \in \mathbb{R}, x > 3$

i) $y = \frac{4}{x-3} \Rightarrow x-3 = \frac{4}{y} \Rightarrow x = \frac{4}{y} + 3 \Rightarrow f^{-1}(x) = \frac{4}{x} + 3$

ii) Range: $g^{-1}(x) > 3$
Domain: $x \in \mathbb{R}$



iv) $\frac{4}{x} + 3 = \frac{4}{x-3} \Rightarrow 4 + 3x = \frac{4x}{x-3}$

$\Rightarrow x-3(4+3x) = 4x$

$\Rightarrow 4x + 3x^2 - 12 - 9x - 4x = 0$

$\Rightarrow 3x^2 - 9x - 12 = 0$

$x = 4$ or $x = -1$

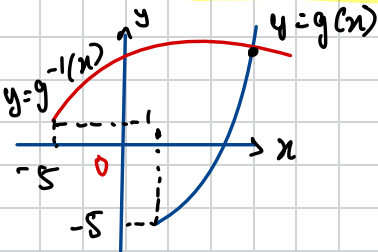
d) $g(x) = (x-1)^2 - 5, x \in \mathbb{R}, x \geq 1$

i) $y = (x-1)^2 - 5 \Rightarrow \sqrt{y+5} + 1 = x \Rightarrow f^{-1}(x) = 1 + \sqrt{x+5}$

ii) Range: $g^{-1}(x) \geq 1$

Domain: $x \geq -5$

iii) $(x-1)^2 - 5 = 1 + \sqrt{x+5}$
 $\Rightarrow (x-1)^2 = 6 + \sqrt{x+5}$
 $\Rightarrow ((x-1)^2 - 6)^2 = (\sqrt{x+5})^2$



$\Rightarrow (x^2 - 2x - 5)^2 = x + 5 \Rightarrow x^4 - 4x^3 - 6x^2 + 20x + 25 = x + 5$

$\Rightarrow x^4 - 4x^3 - 6x^2 + 19x + 20 = 0$

$\Rightarrow x = 4$ OR $x = -2$ OR $x = -1$ OR $x = -7$

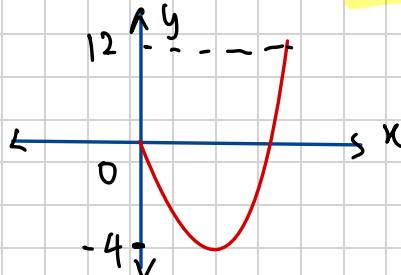
③ $f(x) = \frac{2}{x}, x \in \mathbb{R}, x \neq 0$

a) $f(x) = y \Rightarrow y = \frac{2}{x} \Rightarrow x = \frac{2}{y} \Rightarrow f^{-1}(x) = \frac{2}{x}$, $f(x)$ & f is self inverse

b) $ff(x) \Rightarrow \frac{2}{\frac{2}{x}} = x$

④ $g: x \mapsto x^2 - 4x, x \in \mathbb{R}, 0 \leq x \leq 6$

$g(x)$ is not one to one in the interval $0 \leq x \leq 6$



⑤ $f(x) = \frac{3x-5}{x-3}$ a) $ff(x) = x \Rightarrow 3 \left(\frac{3x-5}{\frac{3x-5}{x-3}} \right) - 5 = \frac{(3x-5)}{\left(\frac{3x-5}{x-3} \right) - 3}$

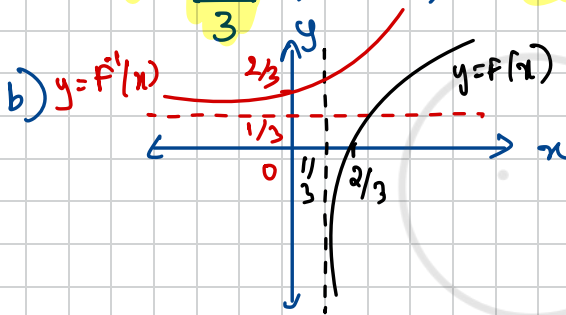
$$\Rightarrow \frac{3(3x-5) - 5(x-3)}{3x-5-3(x-3)} = \frac{9x-15-5x+15}{3x-5-3x+9} = \frac{4x}{4} = x$$

$$b) f^{-1}(x) = \frac{3x-5}{x-3} \quad (\text{As } x=y)$$

$$⑥ f: x \rightarrow \ln(3x-1), \quad x \in \mathbb{R}, \quad x > \frac{1}{3}$$

$$a) y = \ln(3x-1) \Rightarrow e^y = 3x-1 \Rightarrow \frac{e^y+1}{3} = x \Rightarrow f^{-1}(x) = \frac{e^x+1}{3}$$

$$f^{-1}(x) = \frac{e^x+1}{3}; \quad x \in \mathbb{R}, \quad f^{-1}(x) > \frac{1}{3}$$



$$⑦ g(x) = x^2 - 6x + 8, \quad x \in \mathbb{R}, \quad x > 3$$

$$a) y = x^2 - 6x + 8 \Rightarrow y = (x-3)^2 - 1 \Rightarrow x = 3 + \sqrt{y+1}$$

$$g^{-1}(x) = 3 + \sqrt{x+1}; \quad \text{domain: } x \in \mathbb{R}; \quad \text{Range: } g^{-1}(x) > 3$$

$$b) x^2 - 6x + 8 = 3 + \sqrt{x+1} \Rightarrow (x^2 - 6x + 5)^2 = (\sqrt{x+1})^2$$

$$\Rightarrow x^4 - 12x^3 + 46x^2 - 60x + 25 = x+1$$

$$\Rightarrow x^4 - 12x^3 + 46x^2 - 61x + 24 = 0$$

$$\Rightarrow x = \frac{7 + \sqrt{17}}{2}$$

$$⑧ f(x) = \frac{2x+1}{x-3}, \quad x \in \mathbb{R}, \quad x > 3$$

$$a) y = \frac{2x+1}{x-3} \Rightarrow xy - 2x = 3y+1$$

$$x(y-2) = 3y+1$$

$$x = \frac{3y+1}{y-2}$$

$$f^{-1}(x) = \frac{3x+1}{x-2}$$

$$b) f^{-1}(x) = 8$$

$$8 = \frac{3x+1}{x-2}$$

$$\Rightarrow 8x - 16 = 3x + 1$$

$$5x = 17 \quad x = \frac{17}{5}$$