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2.3 - Composite functions

① $f(x) = x^3$, $g(x) = x - 2$, $h(x) = \frac{x}{2}$

a) $fg(4) \Rightarrow g(4) = 4 - 2 = 2 \Rightarrow f(2) = (2)^3 = 8$

b) $gf(2) \Rightarrow f(2) = (2)^3 = 8 \Rightarrow g(8) = 8 - 2 = 6$

c) $g^2(-1) \Rightarrow g(-1) = -1 - 2 = -3 \Rightarrow g(-3) = -3 - 2 = -5$

d) $gh(6) \Rightarrow h(6) = \frac{6}{2} = 3 \Rightarrow g(3) = 3 - 2 = 1$

e) $fgh(10) \Rightarrow h(10) = \frac{10}{2} = 5 \Rightarrow g(5) = 5 - 2 = 3 \Rightarrow f(3) = (3)^3 = 27$

② $p(x) = 3x - 1$, $q(x) = 4 - x^2$, $r = \frac{1}{x}$

a) $pq(x) \Rightarrow 3(4 - x^2) - 1 = 12 - 3x^2 - 1 = 11 - 3x^2$

b) $qr(x) \Rightarrow 4 - \left(\frac{1}{x}\right)^2 = 4 - \frac{1}{x^2}$

c) $p^2(x) \Rightarrow 3(3x - 1) - 1 = 9x - 3 - 1 = 9x - 4$

d) $rp(x) \Rightarrow \frac{1}{3x - 1} = \frac{1}{3x - 1}$

e) $pqr(x) = 4 - \left(\frac{1}{x}\right)^2 = 4 - \frac{1}{x^2} = 3\left(4 - \frac{1}{x^2}\right) - 1 = 12 - \frac{3}{x^2} - 1$

$= \frac{12x^2 - 3 - x^2}{x^2} = \frac{11x^2 - 3}{x^2} = 11 - \frac{3}{x^2}$

③ $f: x \mapsto |4x - 5|$ $\quad g: x \mapsto \frac{x - 1}{2}$

a) $fg(-5) \Rightarrow g(-5) = \frac{-5 - 1}{2} = -3 = f(-3) = |4(-3) - 5| = |-12 - 5| = 17$

b) $|4\left(\frac{x - 1}{2}\right) - 5| = |2x - 2 - 5| = |2x - 7| = 2x \Rightarrow -2x + 7 = 2x$ ($fg(x) = 2x$)

$\Rightarrow 4x = 7 \quad x = \frac{7}{4}$

④ $f(x) = e^x$, $x \in \mathbb{R}$ & $g(x) = 2\ln x$, $x \in \mathbb{R}$, $x > 0$

a) $gf(x) = 2\ln(e^x) = \ln(e^x)^2 = \ln e^{2x} = 2x$

b) $gf(x) = fg(x)$ $\quad gf(x) = 2x \Rightarrow fg(x) = e^{2\ln x} = e^{\ln x^2} = x^2$

$\Rightarrow 2x = x^2 = x^2 - 2x = 0 \Rightarrow x(x-2) = 0 \quad x = 0$ or $x = 2$

As $x > 0$, $x = 2$ is the only one real value.

⑤ $f(x) = 4x^2 - 8x$ $\quad g(x) = k - x$

a) Range of $f(x) \Rightarrow 4(x^2 - 2x) = 4((x-1)^2 - 1^2) = 4(x-1)^2 - 4$

$\Rightarrow f(x) \geq -4$

b) $gf(x) = k - (4x^2 - 8x) = k + 8x - 4x^2$

c) $gf(x) < 6 \Rightarrow k + 8x - 4x^2 < 6 = -4x^2 + 8x + (k-6) < 0$

$b^2 - 4ac < 0 \Rightarrow (-8)^2 - 4(-4)(k-6) < 0 \Rightarrow 64 + 16k - 96 < 0$

$16k < 32 = k < 2 = \{k : k \in \mathbb{R}, k < 2\}$

⑥ $f(x) = 8 - (x-2)^2$

a) Range of $f(x)$: $f(x) \leq 8$

b) $ff(-2) \Rightarrow f(-2) = 8 - (-2-2)^2 = -8 = f(-8) = 8 - (-8-2)^2 = -92$

⑦ $f(x) = 2x^2$ $\quad g(x) = \dfrac{1}{x-1}$

a) Range: $F(x) \geq 0$ $\quad$ b) $fg(x) = 2\left(\dfrac{1}{x-1}\right)^2 = 2\left(\dfrac{1}{(x-1)^2}\right) = \dfrac{2}{(x-1)^2}$

c) $fg(x) = 18 = 2\left(\dfrac{1}{x-1}\right)^2 = 18 \Rightarrow \left(\dfrac{1}{x-1}\right)^2 = 9 = \dfrac{1}{x-1} = \pm 3$

$x - 1 = \pm\dfrac{1}{3}$ $\quad x = \dfrac{1}{3} + 1$ $\quad$ or $\quad x = -\dfrac{1}{3} + 1 \Rightarrow x = \dfrac{2}{3}$ or $x = \dfrac{4}{3}$

⑧ $p: x \longmapsto 3^x$, $q: x \longmapsto x-2$

a) $pq(x) = 3^{x-2}$ b) $qp(x) = 3^x - 2$

c) $pq(x) = qp(x) \Rightarrow 3^x - 2 = 3^{x-2} \Rightarrow 3^x - (3^{x-2}) = 2$

$3^x(1-3^{-2}) = 2 \Rightarrow 3^x\left(1-\frac{1}{9}\right) = 2 \Rightarrow 3^x \times \left(\frac{8}{9}\right) = 2$

$3^x = 2 \times \frac{9}{8_4} \Rightarrow 3^x = \frac{9}{4} = \ln(3^x) = \ln\left(\frac{9}{4}\right)$

$\Rightarrow x \ln 3 = \ln \frac{9}{4}$

$$x = \frac{\ln \frac{9}{4}}{\ln 3}$$