Author: Alina Prynkevych

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13.4 Definite integrals

1) a) $\int_2^{10} x^2\,dx$

$$\left[\frac{x^3}{3}\right]_2^{10} = \left(\frac{(10)^3}{3}\right) - \left(\frac{(2)^3}{3}\right) = \frac{992}{3}$$

b) $\int_1^8 6x^{\frac{1}{3}}\,dx$

$$\left[\frac{6x^{4/3}}{4/3}\right]_1^8 = \left(\frac{6(8)^{4/3}}{4/3}\right) - \left(\frac{6(1)^{4/3}}{4/3}\right) = \frac{135}{2}$$

c) $\int_2^4 4x^{-3}\,dx$

$$\left[\frac{4x^{-2}}{-2}\right]_2^4 = \left(-2(4)^{-2}\right) - \left(-2(2)^{-2}\right) = \frac{3}{8}$$

d) $\int_4^9 5x^{-\frac{1}{2}}\,dx$

$$\left[\frac{5x^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^9 = \left(10(9)^{\frac{1}{2}}\right) - \left(10(4)^{\frac{1}{2}}\right) = 10$$

2) a) $\int_1^2 18x - 4x^{-3}\,dx$

$$\left[\frac{18x^2}{2} - \frac{4x^{-2}}{-2}\right]_1^2 = \left[9x^2 + \frac{2}{x^2}\right]$$

$$\left(9(2)^2 + \frac{2}{2^2}\right) - \left(9(1)^2 + \frac{2}{1^2}\right) = 25.5$$

b) $\int_1^4 x^{-\frac{1}{2}}(14x^3 - 5x^2)\,dx$

$$\int_1^4 14x^{\frac{5}{2}} - 5x^{\frac{3}{2}}\,dx$$

$$\left[4x^{\frac{7}{2}} - 2x^{\frac{5}{2}}\right]_1^4$$

$$\left(4(4)^{\frac{7}{2}} - 2(4)^{\frac{5}{2}}\right) - \left(4(1)^{\frac{7}{2}} - 2(1)^{\frac{5}{2}}\right) = 446$$

c) $\int_1^{36}\left(2x^{1/2} - 6x^{-1}\right)^2 dx$

$4x - \frac{24}{\sqrt{x}} + \frac{36}{x^2}$

$\left[\frac{4x^2}{2} - \frac{24x^{\frac{1}{2}}}{1/2} + \frac{36x^{-1}}{-1}\right]_1^{36}$

$\left(2x^2 - 48x^{1/2} - 36x^{-1}\right]_1^{36}$

$\left(2(36)^2 - 48(36)^{\frac{1}{2}} - 36(36)^{-1}\right) - \left(2(1)^2 - 48(1)^{\frac{1}{2}} - 36(1)^{-1}\right) = 2385$

3) $\int_1^2 5x^{-2} - 3x^{-4}\,dx$

$\left[\frac{5x^{-1}}{-1} - \frac{3x^{-3}}{-3}\right]_1^2 = \left[\frac{-5}{x} + \frac{1}{x^3}\right]_1^2$

$\left(\frac{-5}{2} + \frac{1}{2^3}\right) - \left(\frac{-5}{1} + \frac{1}{1^3}\right) = \frac{13}{8}$

4) $\int_1^{16}\left(2x^{-2} + 4x^{\frac{1}{2}}\right)dx$

$\left[\frac{2x^{-1}}{-1} + \frac{4x^{3/2}}{3/2}\right]_1^{16} = \left[-\frac{2}{x} + \frac{8x^{3/2}}{3}\right]_1^{16}$

$\left(-\frac{2}{x} + \frac{8x^{3/2}}{3}\right) - \left(-\frac{2}{x} + \frac{8x^{3/2}}{3}\right) = \frac{1359}{8}$

5) $\int_1^4\left(\frac{8}{3}x^3 - 6x^{-2} - 2x^{-\frac{1}{2}}\right)dx$

$\left[\frac{\frac{8}{3}x^4}{4} - \frac{6x^{-1}}{-1} - \frac{2x^{\frac{1}{2}}}{1/2}\right]_1^4 = \left[\frac{2}{3}x^4 + 6x^{-1} - 4x^{\frac{1}{2}}\right]_1^4$

$\left(\frac{2}{3}(4)^4 + 6(4)^{-1} - 4(4)^{\frac{1}{2}}\right) - \left(\frac{2}{3}(1)^4 + 6(1)^{-1} - 4(1)^{\frac{1}{2}}\right) = \frac{323}{2}$ or 161.5

6) $\int_4^k 9x^{1/2}\,dx = 702$

$\left[\frac{9x^{3/2}}{3/2}\right]_4^k = \left[6x^{3/2}\right]_4^k$

$\left(6(k)^{3/2}\right) - \left(6(4)^{3/2}\right) = 702$

$$6k^{3/2} - 48 = 702$$

$$6k^{3/2} = 750$$

$$k^{3/2} = 125$$

$$(k^{3/2})^{2/3} = (125)^{2/3}$$

$$k^{1} = 25$$

7) a) $y^{1/2} = 2x^{1/3} - \frac{1}{2}x^{-\frac{1}{3}}$

$$(y^{1/2})^2 = \left(2x^{1/3} - \frac{1}{2}x^{-\frac{1}{3}}\right)^2$$

$$\left(2x^{1/3} - \frac{1}{2}x^{-\frac{1}{3}}\right)\left(2x^{1/3} - \frac{1}{2}x^{-\frac{1}{3}}\right)^2$$

$$y = 4x^{\frac{2}{3}} + \frac{1}{4}x^{-\frac{3}{2}} - 2$$

b) $\int_1^8 4x^{\frac{2}{3}} + \frac{1}{4}x^{-\frac{3}{2}} - 2\,dx$

$$\left[\frac{4x^{5/3}}{5/3} + \frac{\frac{1}{4}x^{\frac{1}{3}}}{1/3} - 2x\right]_1^8 = \left[\frac{12x^{5/3}}{5} + \frac{3x^{\frac{1}{3}}}{4} - 2x\right]_1^8$$

$$\left(\frac{12(8)^{5/3}}{5} + \frac{3(8)^{\frac{1}{3}}}{4} - 2(8)\right) - \left(\frac{12(1)^{5/3}}{5} + \frac{3(1)^{\frac{1}{3}}}{4} - 2(1)\right) = \frac{1223}{20}$$

8) $\int_4^9 ax^{1/2}\,dx = -114$

$$\left[\frac{ax^{3/2}}{3/2}\right]_4^9 = \left[\frac{2ax^{3/2}}{3}\right]_4^9$$

$$\left(\frac{2a(9)^{3/2}}{3}\right) - \left(\frac{2a(4)^{3/2}}{3}\right) = -114$$

$$18a - \frac{16}{3}a = -114$$

$$a\left(18 - \frac{16}{3}\right) = -114$$

$$a = \frac{-114}{38/3}$$

$$a = -9$$