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13.2 Indefinite integrals

1) a) $\int x^{4+1} dx$

$$\int \frac{x^5}{5} + c$$

$$= \frac{1}{5} x^5 + c$$

b) $\int -7x^{6+1} dx$

$$\int \frac{-7x^7}{7} + c$$

$$= -x^7 + c$$

c) $\int 4x^{-2+1} dx$

$$\int \frac{4x^{-1}}{-1} + c$$

$$= -4x^{-1} + c$$

2) a) $\int 5x^2 + 9x^{-3} dx$

$$\int 5x^2 + \int 9x^{-3}$$

$$\int \frac{5x^3}{3} + \frac{9x^{-2}}{-2} + c$$

$$= \frac{5}{3} x^3 - \frac{9}{2} x^{-2} + c$$

b) $\int \left(\frac{3}{4} x^5 - 4x^{-5} + 8x \right) dx$

$$\int \left(\frac{3}{4} x^5 \right) - \int 4x^{-5} + \int 8x$$

$$\frac{\frac{3}{4} x^6}{6} - \frac{4x^{-4}}{-4} + \frac{8x^2}{2} + c$$

$$= \frac{1}{8} x^6 - x^{-4} + 4x^2 + c$$

c) $\int (4x^{3/2} - 10x^{-3/2}) dx$

$$\int 4x^{3/2} - \int 10x^{-3/2}$$

$$\frac{4x^{5/2}}{5/2} - \frac{10x^{-1/2}}{-1/2} + c$$

$$= \frac{8}{5} x^{5/2} + 20x^{-1/2} + c$$

3) a) $\int (6-5\sqrt{x})^2 dx$

$$(6-5\sqrt{x})(6-5\sqrt{x})$$

$$36 - 30\sqrt{x} - 30\sqrt{x} + 25x$$

$$\int = 25x - 60\sqrt{x} + 36$$

~~$$\frac{25x^2}{2}$$~~

$$\int 25x - 60x^{1/2} + 36$$

$$= \frac{25x^2}{2} - \frac{60x^{3/2}}{3/2} + 36x + c$$

$$\frac{25}{2} x^2 - 40x^{3/2} + 36x + c$$

b) $\int \left(\frac{3}{4} x^5 - 4x^{-5} + 8x \right) dx \Rightarrow \int \frac{(3x-2)^2}{\sqrt{x}} dx$

$$\frac{(3x-2)^2}{\sqrt{x}} = \frac{9x^2 - 12x + 4}{x^{1/2}}$$

$$\int 9x^{3/2} - 12x^{1/2} + 4x^{-1/2} \rightarrow \frac{9x^{5/2}}{5/2} - \frac{12x^{3/2}}{3/2} + \frac{4x^{1/2}}{1/2}$$

$$= \frac{18}{5} x^{5/2} - 8x^{3/2} + 8x^{1/2} + c$$

c) $\int \left(\frac{5-7x^3+4x^{-3/2}}{x^2} \right) dx$

$$\int \left(\frac{5}{x^2} - \frac{7x^3}{x^2} + \frac{4x^{-3/2}}{x^2} \right) dx$$

$$\int (5x^{-2} - 7x + 4x^{-7/2})$$

$$\frac{5x^{-1}}{-1} - \frac{7x^2}{2} + \frac{4x^{-5/2}}{5/2} + c$$

$$-5x^{-1} - \frac{7}{2} x^2 - \frac{8}{5} x^{-5/2} + c$$

4) a) $\int \left(\frac{P}{\sqrt{x}} - R x^{3/2} \right) dx$

$$= \frac{P}{x^{1/2}} - R x^{3/2} \rightarrow \int P x^{-1/2} - R x^{3/2}$$

$$= \frac{P x^{1/2}}{1/2} - \frac{R x^{5/2}}{5/2} + c$$

$$= 2P x^{1/2} - \frac{2}{5} R x^{5/2} + c$$

b) $\int \left(\frac{A}{x^3} + \frac{B}{x^5} \right) dx$

$$\int A x^{-3} + B x^{-5}$$

$$= \frac{A x^{-2}}{-2} + \frac{B x^{-4}}{-4} = -\frac{1}{2} A x^{-2} - \frac{1}{4} B x^{-4} + c$$

c) $\int \left(M \sqrt[3]{x} - N x^{3/4} \right) dx \rightarrow \int M x^{1/3} - N x^{3/4}$

$$= \frac{M x^{4/3}}{4/3} - \frac{N x^{7/4}}{7/4} + c \rightarrow \frac{3}{4} M x^{4/3} - \frac{4}{7} N x^{7/4} + c$$

$$5) (y^{1/2} = 6x^{1/5} - 5)^2$$

$$y = (6x^{1/5} - 5)^2$$

$$y = 36x^{2/5} - 60x^{1/5} + 25$$

$$\int y = 36x^{2/5} - 60x^{1/5} + 25$$

$$\frac{36x^{7/5}}{7/5} - \frac{60x^{6/5}}{6/5} + 25x + c$$

$$\frac{180}{7} x^{7/5} - 50x^{6/5} + 25x + c$$

$$6) g(x) = 3x^3 - 7\sqrt{x} + \frac{4}{\sqrt[3]{x}} - \frac{2}{x^2}$$

$$\int g(x) = 3x^3 - 7x^{1/2} - 4x^{-1/3}$$

$$= \frac{3x^4}{4} - \frac{7x^{3/2}}{3/2} - \frac{4x^{2/3}}{2/3} + a - 2x^{-2} + c$$

$$= \frac{3}{4} x^4 - \frac{14}{3} x^{3/2} - 6x^{2/3} - \frac{2x^{-1}}{-1} + c$$

$$= \frac{3}{4} x^4 - \frac{14}{3} x^{3/2} - 6x^{2/3} + 2x^{-1} + c$$

$$7) \int (abx^2 + \frac{4b}{x^2}) dx = 2x^3 - \frac{2}{x^4} + c$$

$$\frac{abx^3}{3} - \frac{b}{x^4} + c$$

$$= 2x^3 - 2x^{-4} + c \rightarrow \textcircled{1} \frac{ab}{3} = 2$$

$$\textcircled{2} b = 2$$

$$\frac{a(2)}{3} = 2$$

$$2a = 6$$

$$a = 3$$

$$8) a) g(x) = (1+2x)(4-3x)^8 \rightarrow 4^8 + {}^8C_1(4)^7(3x) + {}^8C_2(4)^6(3x)^2$$

$$g(x) = 65536 - 393216x + 1032192x^2$$

$$b) \int g(x) = 65536x - 131072x^2$$

$$\int g(x) = 65536x - 131072x^2 + 81920x^3 + c$$