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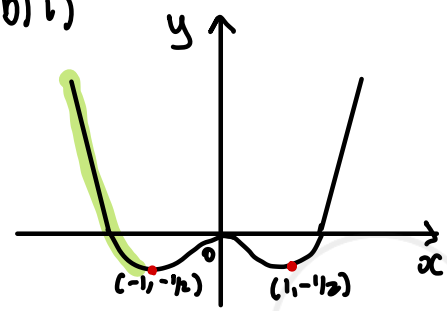
12.7 Increasing and Decreasing Functions

1) a) $f(x) = \frac{1}{2}x^4 - x^2$

$y = f(x) = 0$

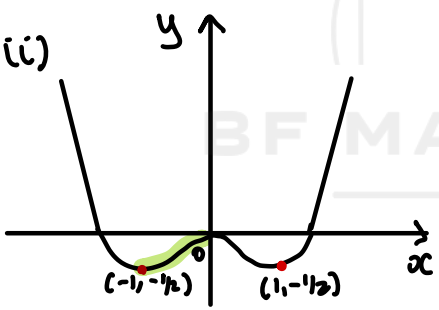
$(-1, -1/2)\ (0, 0)\ (1, -1/2)$

b) i)



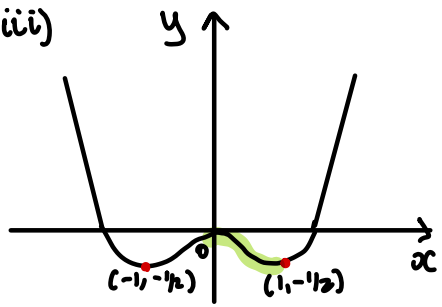
$(-\infty, -1)$ = decreasing because -ve gradient

ii)



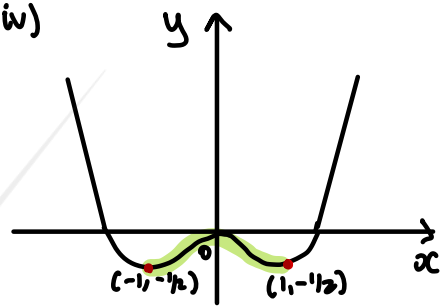
$(-1, 0)$ = increasing because +ve gradient

iii)



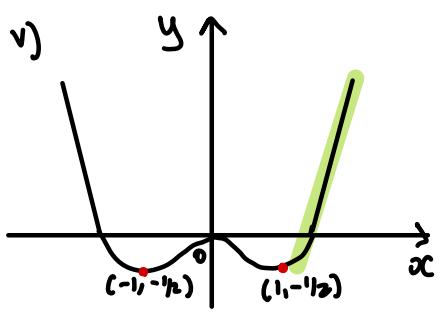
$(0, 1)$ = decreasing because -ve gradient

iv)



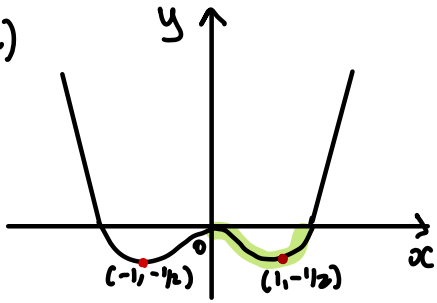
$(-1, 1)$ = neither because it is both +ve and -ve gradient.

v)



$(1, \infty)$ = increasing because +ve gradient

vi)



(0,2) = neither because +ve and -ve gradient.

c) $f(x) = x^3 - 12x +$

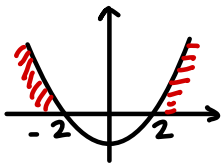
$f'(x) = 3x^2 - 12$

$3x^2 - 12 \geq 0$

$3x^2 \geq 12$

$x^2 \geq 4$

$x \geq \pm\sqrt{4}$



$x \leq -2 \quad x \geq 2$

2) a) $f(x) = 5x^2 - 8x + 7$

$f'(x) = 10x - 8$

$10x - 8 \geqslant 0$

$10x \geqslant 8$

$x \geqslant \frac{8}{10}$

$x \geqslant 0{\cdot}8$

b) $f(x) = 14 - 9x - 3x^2$

$f'(x) = -6x - 9$

$-6x - 9 \geqslant 0$

$-6x \geqslant 9$

$x \leqslant \frac{9}{-6}$

$x \leqslant -\frac{3}{2}$

d) $f(x) = 18x - 2x^3$

$f'(x) = -6x^2 + 18$

$-6x^2 + 18 \geqslant 0$

$-6x^2 \geqslant -18$

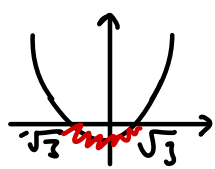
$6x^2 \leqslant 18$

sign flips because $\times -1$ both sides

$x^2 \leqslant 3$

$x = \pm\sqrt{3}$

$x^2 - 3 \leqslant 0$



$-\sqrt{3} \leqslant x \leqslant \sqrt{3}$

3) a) $f(x) = 3x^2 + 7x - 2$

$f'(x) = 6x + 7$

$6x + 7 \leqslant 0$

$6x \leqslant -7$

$x \leqslant -\frac{7}{6}$

b) $f(x) = 15 - 6x - 5x^2$

$f'(x) = -6 - 10x$

$-6 - 10x \leq 0$

$-10x \leq 6$

$x \geq -\frac{6}{10} \rightarrow -\frac{3}{5}$

$x \geq -\frac{3}{5}$

4) $f(x) = x^5 + 2x^3 + 8x - 18$

$f'(x) = 5x^4 + 6x^2 + 8$

$\therefore 5x^4 \geq 0$ and $6x^2 \geq 0$

$\therefore 5x^4 + 6x^2 + 8 > 0$

$\therefore f'(x) > 0$

$\therefore f(x)$ is increasing for all values of x.

c) $f(x) = -2x^5 - 8x^3$

$f'(x) = -10x^4 - 24x^2$

$-10x^4 - 24x^2 \leq 0$

$10x^4 + 24x^2 \geq 0$

$2x^2(2x^2 + 12) \geq 0$

$2x^2 \geq 0$ $\quad 2x^2 + 12 \geq 0$

$x \geq 0$ $\quad 2x^2 \geq -12$

$x^2 \geq -6$

no solution

5) $g(x) = -2x^3 - 18x^2 - 54x +$

$g'(x) = -6x^2 - 36x - 54$

$-6x^2 - 36x - 54 \leq 0$

$\hookrightarrow -6(x+3)^2 \leq 0$ for all values of x.

d) $f(x) = 5x(3 - x)$

$f(x) = 15x - 5x^2$

$f'(x) = 15 - 10x$

$15 - 10x \leq 0$

$15 \leq 10x$

$x \geq 1.5$

6) $h(x) = x^3 + qx$

$h'(x) = 3x^2 + q$

$3x^2 + q \geq 0$

$q \geq 0$

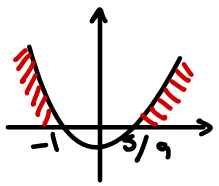
7) a) $y = 3x^3 + 2x^2 - 5x - 6$

$\frac{dy}{dx} = 9x^2 + 4x - 5$

b) $9x^2+4x-5 \geq 0$

$(x+1)(9x-5) \geq 0$

$x=-1 \quad x=\frac{5}{9}$



$x \leq -1$

$x \geq \frac{5}{9}$

$\{x: x \leq -1\} \cup \{x: x \geq 5/9\}$