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12.6

1) a) $\frac{dy}{dx} = 3x^2 - 8x$

$\rightarrow 3(2)^2 - 8(2) = -4$

$y + 8 = -4(x - 2)$

$y + 8 = -4x + 8$

$y = -4x$

b) $y = 4x^{-1} - 3x^2$

$\frac{dy}{dx} = -4x^{-2} - 6x$

$\rightarrow -4(-2)^{-2} - 6(-2) = 11$

$y + 14 = 11(x + 2)$

$y + 14 = 11x + 22$

$y = 11x + 8$

c) $\frac{dy}{dx} = 9x^{1/2} - 2x^{-1/2}$

$\rightarrow 9(4)^{1/2} - 2(4)^{-1/2}$

$= 17$

when $x = 4$

$y = 6(4)^{3/2} - 4(4)^{1/2} = 40$

$y - 40 = 17(x - 4)$

$y - 40 = 17x - 68$

$y = 17x - 28$

d) $y = \frac{12}{2x} - \frac{x^2}{2x} \rightarrow \frac{6}{x} - \frac{x}{2} \rightarrow 6x^{-1} - \frac{1}{2}x$

$\frac{dy}{dx} = -6x^{-2} - \frac{1}{2}$

$-6(-3)^{-2} - \frac{1}{2}$

$= -\frac{7}{6}$

when $x = -3$

$y = \frac{12 - (-3)^2}{2(-3)} = -\frac{1}{2}$

$y + \frac{1}{2} = -\frac{7}{6}(x + 3)$

$y + \frac{1}{2} = -\frac{7}{6}x - \frac{7}{2}$

$y = -\frac{7}{6}x - 4$

2) a) $y = \frac{1}{2}x^2 + 6x^{-1}$

$\frac{dy}{dx} = x - 6x^{-2}$

when $x = 3$

$3 - 6(3)^{-2} = \frac{7}{3}$ $\therefore$ gradient of normal $= -\frac{3}{7}$

$y - \frac{13}{2} = -\frac{3}{7}(x - 3)$

$y - \frac{13}{2} = -\frac{3}{7}x + \frac{9}{7}$

$y = -\frac{3}{7}x + \frac{109}{14}$

b) $y = 4x^{-1} - 5x^2$

$\frac{dy}{dx} = -4x^{-2} - 6x$

↳ $-4(4)^{-2} - 6(4)$

$= -97/4$ → gradient of normal $= 4/97$

when $x = 4$

$y = 4/4 - 5(4)^2 = -47$

$y + 47 = 4/97 (x - 4)$

$y + 47 = 4/97\, x - 16/97$

$y = 4/97\, x - 4575/97$

3) a) $\frac{dy}{dx} = -2 - 2x$

↳ $-2 - 2(1)$

$= -4$ → gradient

when $x = 1$

$y = 10 - 2(1) - 1(1)^2$

$= 7$

$y - 7 = -4(x - 1)$

$y - 7 = -4x + 4$

$y = -4x + 11$

b) gradient $= 1/4$

$y - 7 = 1/4 (x - 1)$

$y - 7 = 1/4\, x - 1/4$

$y = 1/4\, x + 27/4$

4) a) $\frac{dy}{dx} = 2x - 5$

↳ $2(5) - 5$

$= 5$

when $x = 5$

$y = (5)^2 - 5(5) + 4$

$= 4$

$y - 4 = 5(x - 5)$

$y - 4 = 5x - 25$

$y = 5x - 21$

b) $\frac{dy}{dx} = 2x - 5$

↳ $2(3) - 5$

$= 1$

↳ -1 ∴ gradient of normal

when $x = 3$

$y = 3^2 - 5(3) + 4$

$= -2$

$y + 2 = -1(x - 3)$

$y + 2 = -x + 3$

$y = -x + 1$

c) $5x - 21 = -x + 1$

$6x = 22$

$x = 11/3$

$y = -(11/3) + 1$

$= -8/3$

∴ $\left(\frac{11}{3}, -\frac{8}{3}\right)$

5) a) $g'(x) = 3x^2 - 4$

$\therefore 3(-1)^2 - 4 = -1$

when $x = -1$

$y = (-1)^3 - 4(-1) = 3$

$y - 3 = -1(x+1)$

$y - 3 = -x - 1$

$y = -x + 2$

b) when $y = 0$

$0 = -x + 2$ $\quad \therefore A(2,0)$

$x = 2$

c) when $x = 2$

$y = (2)^3 - 4(2) = 0$

$\therefore$ A lies on the curve

6) when $x = 6$

$y = \frac{1}{2}(6)^2 - 8(6) + 6 = -24$

$\therefore P(6, -24)$

$\frac{dy}{dx} = x - 8$

$\therefore 6 - 8 = -2$

$\therefore$ normal $= \frac{1}{2}$

$y + 24 = \frac{1}{2}(x - 6)$

$y + 24 = \frac{1}{2}x - 3$

$y = \frac{1}{2}x - 27$

$\frac{1}{2}x - 27 = \frac{1}{2}x^2 - 8x + 6$

$x^2 - 17x + 66 = 0$

$(x - 6)(x - 11) = 0$

$x = 6$ $\quad x = 11$

$\downarrow$ point P $\quad \downarrow$ point Q

$\therefore Q\left(11, -\frac{43}{2}\right)$

when $x = 11$

$y = \frac{1}{2}(11) - 27 = -43/2$

7) a) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

$y = x^{1/2} + x^{-1/2}$

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$

$= \frac{1}{2}(4)^{-1/2} - \frac{1}{2}(4)^{-3/2}$

$= \frac{3}{16}$

$y - 2.5 = \frac{3}{16}(x - 4)$

$y - 2.5 = \frac{3}{16}x - \frac{3}{4}$

$y = \frac{3}{16}x + \frac{7}{4}$ (x16)

$16y = 3x + 28$

$3x - 16y + 28 = 0$

b) Gradient of normal $\rightarrow -\frac{16}{3}$

$y - 2.5 = -\frac{16}{3}(x - 4)$

$3y - 7.5 = -16x + 64$

$16x + 3y - 71.5 = 0$ (x2)

$32x + 6y - 143 = 0$

c) when $x = 0$

$3(0) - 16y + 28 = 0$ A $(0, \frac{7}{4})$

$-16y + 28 = 0$

$28 = 16y$

$y = \frac{7}{4}$

when $x = 0$

$32(0) + 6y - 143 = 0$ B $(0, \frac{143}{6})$

$6y - 143 = 0$

$6y = 143$

$y = \frac{143}{6}$

$\frac{1}{2} \times \left(\frac{143}{6} - \frac{7}{4}\right) \times 4$

$= \frac{265}{6}$