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11.5 solution Bank

(1) a) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$
$= q - p$

b) $\overrightarrow{PN} = 1/4\,\overrightarrow{PQ}$

c) $\overrightarrow{ON} = \overrightarrow{OP} + \overrightarrow{PN}$

d) $p + 1/4\,q - 1/4\,p$
$= 3/4\,p + 1/4\,q$

(2) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= b - a$
$\overrightarrow{AM} = 2/5\,(b-a) = 2/5\,b - 2/5\,a$
$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$
$= a + 2/5\,b - 2/5\,a$
$= 3/5\,a + 2/5\,b$

(3) $\overrightarrow{XD} = \overrightarrow{XA} + \overrightarrow{AD} = -3/4\,\overrightarrow{AB} + \overrightarrow{AD}$
$= -3/4\,a + b$
$\overrightarrow{XY} = 2/5\,\overrightarrow{XD} = 2/5\,(-3/4\,a + b)$
$= -3/10\,a + 2/5\,b$

(4) $\overrightarrow{PQ} \cdot \overrightarrow{PR} = (8i + j) \cdot (2i - 6j)$
$= 16 - 6 = 10$
$|\overrightarrow{PQ}| = \sqrt{8^2 + 1^2}$
$= \sqrt{65}$
$|\overrightarrow{PR}| = \sqrt{2^2 + (-6)^2}$
$= \sqrt{40}$
$\dfrac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|} = \dfrac{10}{\sqrt{65}\sqrt{40}}$
$= \dfrac{10}{\sqrt{260}}$
$= \dfrac{1}{\sqrt{26}}$
$\angle QPR = \cos^{-1}\left(\frac{1}{\sqrt{26}}\right) = 78.7°$

(5) a) $\overrightarrow{KL} = \overrightarrow{JL} - \overrightarrow{JK} = b - a$
$\overrightarrow{PR} = 1/2\,(b-a)$
$= 1/2\,\overrightarrow{KL}$
$\overrightarrow{JP} = 1/2\,\overrightarrow{JK} = 1/2\,a$
$\overrightarrow{JR} = 1/2\,\overrightarrow{JL} = 1/2\,b$
$\overrightarrow{PR} = \overrightarrow{JR} - \overrightarrow{JP}$
$= 1/2\,b - 1/2\,a$
$\overrightarrow{KL} = b - a$, $\overrightarrow{PR} = 1/2\,b - 1/2\,a$. $\overrightarrow{PR} = 1/2\,\overrightarrow{KL}$

b) any two vectors where one is scalar multiple of the other, are parallel.
$\overrightarrow{PR}$ and $\overrightarrow{KL}$ are parallel vectors
$\overrightarrow{PR}$ scalar multiple of $\overrightarrow{KL}$

(6) a) i) $\overrightarrow{WY} = \overrightarrow{AY} - \overrightarrow{AW}$
$= \overrightarrow{AD} + \overrightarrow{DY} - \overrightarrow{AW}$
$= b + \frac{2}{3}\overrightarrow{DC} - \frac{1}{3}\overrightarrow{AB}$
$= b + \frac{2}{3}a - \frac{1}{3}a$
$= \frac{1}{3}a + b$

ii) $x = \frac{1}{2}B + \frac{1}{2}C$
$z = \frac{1}{2}A + \frac{1}{2}D$
$x = \frac{1}{2}(Aa) + \frac{1}{2}(A + b + a)$
$= A + a + \frac{1}{2}b$
$z = \frac{1}{2}A + \frac{1}{2}(A + b)$
$= A + \frac{1}{2}b$
$\overrightarrow{xz} = z - x$
$= \left(A + \frac{1}{2}b\right) - \left(A + a + \frac{1}{2}b\right)$
$xz = -a$

6b) $\overrightarrow{AW} = 1/3\,\overrightarrow{AB} = 1/3\,a$
$\overrightarrow{CD} = \overrightarrow{BA} = -a$
$\overrightarrow{DY} = 2/3\,\overrightarrow{DC} = 2/3\,(-a) = -2/3\,a$
$\overrightarrow{AY} = \overrightarrow{AD} + \overrightarrow{DY} = b - \frac{2}{3}a$
$\overrightarrow{WY} = \overrightarrow{AY} - \overrightarrow{AW} = (b - 2/3\,a) - 1/3\,a$
$= b - a$
$\overrightarrow{WM} = \lambda\overrightarrow{WY} = \lambda(b - a)$
$\overrightarrow{AM} = \overrightarrow{AW} + \overrightarrow{WM}$
$= 1/3\,a + \lambda\,(b + 1/3\,a)$
$= 1/3\,(1 + \lambda)\,a + \lambda b$

c) $\overrightarrow{AX} = 1/2\,a + 1/2\,b$ $\quad \overrightarrow{AZ} = 1/2\,b$
$\overrightarrow{XZ} = \overrightarrow{AZ} - \overrightarrow{AX} = 1/2\,b - (1/2\,a + 1/2\,b) = -1/2\,a$
$\overrightarrow{XM} = \mu + \overrightarrow{Z} = \mu(-1/2\,a) = -1/2\,\mu a$
$\overrightarrow{AM} = \overrightarrow{AX} + \overrightarrow{XM} = (1/2\,a + 1/2\,b) + (-1/2\,\mu a)$
$= (1/2 - 1/2\,\mu)\,a + 1/2\,b$
$= (1 - \mu)\,a + 1/2\,b$
$= a + 1/2\,b - \mu a = (1 - \mu)\,a + 1/2\,b$

6d) $\overrightarrow{WY} = \overrightarrow{AY} - \overrightarrow{AW} = \overrightarrow{AD} + \overrightarrow{DY} - \overrightarrow{AW} = b + \frac{2}{3}\overrightarrow{DC} - \frac{1}{3}\overrightarrow{AB}$

$= b + \frac{2}{3}a - \frac{1}{3}a$

$= b + \frac{1}{3}a$

$\overrightarrow{XZ} = \overrightarrow{AZ} - \overrightarrow{AX} = \frac{1}{2}\overrightarrow{AD} - \overrightarrow{AB} - \frac{1}{2}\overrightarrow{BC} = \frac{1}{2}b - a - \frac{1}{2}a$

$= -a + \frac{1}{2}b - \frac{1}{2}a$

$= -\frac{3}{2}a + \frac{1}{2}b$

$\overrightarrow{WM} = \lambda \overrightarrow{WY}$

$\overrightarrow{AM} = \overrightarrow{AW} + \overrightarrow{WM} = \frac{1}{3}a + \lambda(b + \frac{1}{3}a) = \frac{1}{3}(1+\lambda)a + \lambda b$

$\overrightarrow{XM} = \mu \overrightarrow{XZ}$

$\overrightarrow{AM} = \overrightarrow{AX} + \overrightarrow{XM} = a + \frac{1}{2}b + \mu(-a + \frac{1}{2}b) = (1-\mu)a + \frac{1}{2}(1+\mu)b$

$\frac{1}{3}(1+\lambda)a + \lambda b = (1-\mu)a + \frac{1}{2}(1+\mu)b$

$\lambda = \frac{1}{2}$

$\mu = \frac{1}{2}$

$\overrightarrow{WM} = \frac{1}{2}\overrightarrow{WY}$

$\overrightarrow{XM} = \frac{1}{2}\overrightarrow{XZ}$ $\therefore$ M is the midpoint of WY and XZ

7 a) P divides OA $\rightarrow 1:k$

Q divides AB $\rightarrow 1:k$

$\overrightarrow{OP} = \frac{1}{1+k}\overrightarrow{OA}$ $\quad$ $\overrightarrow{AQ} = \frac{1}{1+k}\overrightarrow{AB}$

$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ} = -\frac{k}{1+k}\overrightarrow{OA} + \frac{1}{1+k}\overrightarrow{AB}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{PQ} = -\frac{k}{1+k}\overrightarrow{OA} + \frac{1}{1+k}(\overrightarrow{OB} - \overrightarrow{OA})$

$= \frac{1}{1+k}\overrightarrow{OB} - \frac{1+k}{1+k}\overrightarrow{OA} = \frac{1}{1+k}\overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{PQ} = -\frac{k}{1+k}\overrightarrow{OA} - \frac{k}{1+k}\overrightarrow{BA}$

$= \frac{k}{1+k}\overrightarrow{OB}$ $\therefore$ $\overrightarrow{PQ}$ scalar multiple of $\overrightarrow{OB}$ vectors are always parallel

b) $\overrightarrow{PQ} = \frac{1}{3}\overrightarrow{OB}$ $\quad$ $\overrightarrow{OB} = b$

$\overrightarrow{PQ} = \frac{1}{3}b$

$\overrightarrow{PQ} = \frac{k}{1+k}\overrightarrow{OB}$

$\frac{k}{1+k}b = \frac{1}{3}b$

$\frac{k}{1+k} = \frac{1}{3}$

$3k = 1 + k$

$3k - k = 1$

$2k = 1$

$k = \frac{1}{2}$